The Use of CAS for Logical Analysis in Mathematics Education

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In mathematics education, ascertaining the concepts of given domains in terms of conceptual and procedural knowledge is essential as a mechanism of knowledge change during knowledge acquisition. Conceptual knowledge consists of an implicit or explicit system of interlinked pieces of knowledge for a given domain, and procedural knowledge comprises systems of multiple execution series for problem solution [1], [2]. The concept of ratio is applied in ascertaining the relation between two quantities and in comparing the relative quantities of two sets. It differs in meaning from simple multiplication and is active in the sense of comparing the relative sizes of given quantities and base quantities rather than directly comparing quantities [3]. So, we focused on comparison of the relations between quantities in two different sets. It has been noted that the concept of ratio can be investigated in a fairly pure form as a logical mathematical recognition [4], and we treated this comparison as a probabilistic comparison task. Ratio and probability are different concepts, but for children unschooled in probability, the ratio concept can be utilized as an approach for probability settings. We consider these problems using theorem prover.

Computer algebra systems are being used frequently in mathematics education. Although good teaching examples and experiences exist, it is clear that the efficient and successful use is not self-evident. A subtle relationship exists between paper-and-pencil techniques, computer algebra systems, and conceptual understanding. The nature of computer systems is different from that of paper-and-pencil techniques. Using a computer algebra system requires insight into procedures as well as into the concepts involved. In addition, the way computer manages the procedures can affect mathematical concepts.

Use of a computer algebra system with a theorem prover can correct the weakness in mathematics education. We can clearly understand mathematical concepts and can minimize the burden of operation opportunities. Computer algebra systems using a theorem prover bring about serious changes in mathematics education. A theorem prover *Theorema* uses a computer algebra system, but reduces the operations of the computer algebra system. Therefore, learners can concentrate on mathematical problems.

In this study, we applied propositional and predicate logic for mathematical

explication of the processes of inference by using theorem prover *Theorema* [5] is a Mathematica package (a collection of packages, in fact) that provides a mathematical assistant system (MAS) inside of and based on Mathematica. A MAS supports the user in formulating and structuring of mathematical knowledge, proving mathematical statements, formulating and executing mathematical algorithms, performing computations, etc. *Theorema* allows you to express algorithms using the language of predicate logic or parts of the Mathematica programming language supporting procedural programming, most importantly various forms of loops. *Theorema* allows you to organize mathematical knowledge as hierarchies of interdependent theories. By settling the definitions, we can continue and try to prove some properties of the newly defined entities.

When considering a human model, the model must be viewed as a process model, as a knowledge model, and as a control model. We can consider that such an approach has been applied to the understanding of theorem proving. A computer algebra system with a theorem prover is being developed. We must consider the theorem prover from mathematical studies. This research aims at extending current computer systems using facilities for supporting mathematical proving. The system consists of a general higher-order predicate logic prover and a collection of special provers. The individual provers imitate the proof style of human mathematicians and produce human-readable proofs in natural language presented in nested cells.

To do mathematics is gaining knowledge and solving problems by reasoning. Theorem prover *Theorema* is a powerful tool for learning mathematics.

References

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