# Some remarks on Taylor's polynomials visualization using Mathematica in context of function approximation.

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In this paper the authors critically analyse popular way of graphic presentation Taylor's polynomials in context of function approximation. They discuss the difficulties of presentation the best local polynomial approximation of function by Taylor's polynomials. Proposed by the authors method of graphical presentation based on table of function and Taylor's polynomials values in neighbourhood of a chosen point. For graphical presentation ListPlot and Plot functions with logarithmic scale in Mathematica System is used.

## Introduction

Taylor's theorem is one of the most classic results of university course in calculus or mathematical analysis. For the case of one variable function $y = f(x)$ and point $x = x_0$, Taylor's polynomial of the $n$-th order is defined as:

$$T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

where a function $f(x)$ have at a point $x_0$ finite derivatives up to the $n$-th order inclusively. Many academic books e.g. [1, 2, 3, 4] contain graphs presented Taylor's polynomials for some elementary functions. For example for $f(x) = e^x$ or $f(x) = \sin x$ as is shown in Figures 1,2. Often these graphs are presented with comments that it shows how well these polynomials approximate $y = f(x)$ near a point $x = x_0$ when $n$ increases.

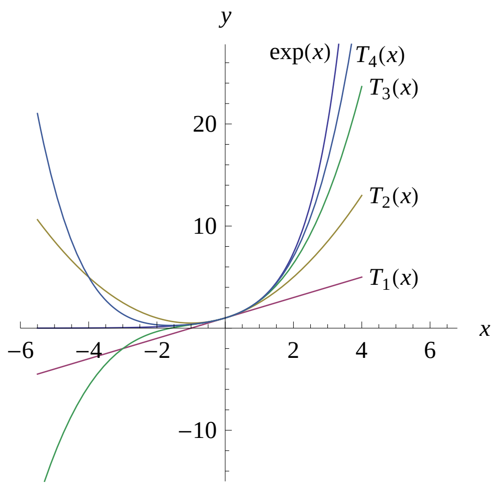


Figure 1: Taylor's polynomials $T_1(x)$, $T_2(x)$, $T_3(x)$, $T_4(x)$ for function $f(x) = e^x$ at point $x_0 = 0$.

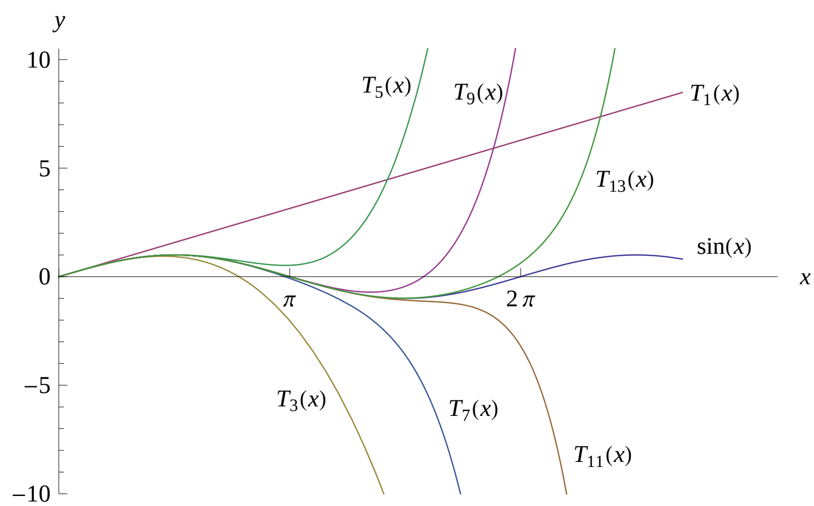


Figure 2: Taylor's polynomials $T_1(x), T_3(x), \ldots, T_{13}(x)$ for function $f(x) = \sin x$ at point $x_0 = 0$.

## 1 Visualization of Taylor's polynomials in context of function approximation

Visualization of Taylor's polynomials is easy and comfortable using CAS packages such as Mathematica, Maple, Derive or others. For a one variable function Mathematica package contains standard procedure Series$[f, \{x, x0, n\}]$ which generates Taylor's polynomial of the $n$-th order for the function $f(x)$ and point $x = x_0$. Using procedure Plot$[\{f_1, f_2, \ldots, f_k\}, \{x, x_{min}, x_{max}\}]$ we can present graphs function $f(x)$ and some its Taylor's polynomials as is shown in Figures 1, 2. But this kind of presentation can be misleading for students in context of the function $f(x)$ approximation by Taylor's polynomials if we do not emphasize the fact of local character of this approximation. In Figures 3, 4 we see that graph of the function $f(x)$ and graphs of Taylor's polynomials seem to overlap close point $x = x_0$. On the base of these Figures we cannot settle which Taylor's polynomial better approximates the function close to the point $x_0$.

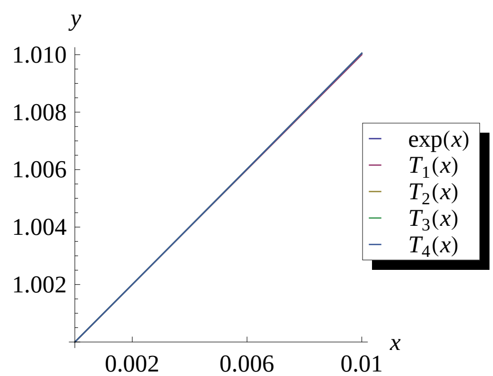


Figure 3: function $f(x) = e^x$ and its Taylor's polynomials $T_1(x)$, $T_2(x)$, $T_3(x)$, $T_4(x)$ in the reduced right neighbourhood $(0, 0.01)$ of the point $x_0 = 0$.

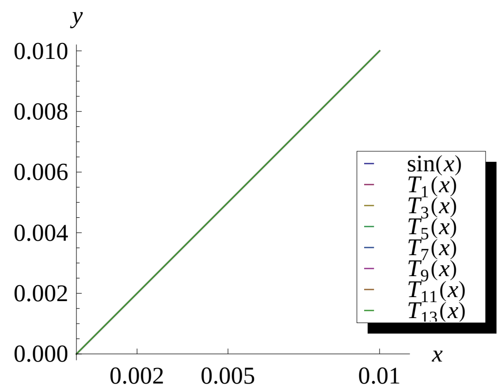


Figure 4: function $f(x) = \sin x$ and its Taylor's polynomials $T_1(x), T_3(x), \ldots, T_{13}(x)$ in the reduced right neighbourhood $(0, 0.01)$ of the point $x_0 = 0$.

In Figures 1, 2 we see that graph of the function $f(x)$ and graphs of Taylor's polynomials seem to overlap close the point $x_0 = 0$. Next, Taylor's polynomials separate from the graph of the $f(x)$. Closer to the point $x_0$ separates Taylor's polynomial of lower order, further from the point $x_0$ separates Taylor's polynomial of higher order. Figures 1, 2 may suggest that overall Taylor's polynomial of higher order better approximates the function than Taylor's polynomial of lower order. But for example, for the function $f(x) = e^x$, $x_0 = 0$ and the point $x = -4$ it is easy to check that:
$T_2(x) = 1 + x + \dfrac{1}{2!}x^2, T_3(x) = 1 + x + \dfrac{1}{2!}x^2 + \dfrac{1}{3!}x^3$, $|f(-4) - T_2(-4)| = |e^{-4} - 5| < |f(-4) - T_3(-4)| = |e^{-4} + 17/3|$. So, $T_2(x)$ better approximates the function $f(x) = e^x$ at the point $x = -4$ than $T_3(x)$. Similarly, for the function $f(x) = \sin x$, $x_0 = 0$ and the point $x = \frac{5}{4}\pi$ we have:
$T_3(x) = x - \frac{1}{3!}x^3, T_5(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$, and $|f(\frac{5}{4}\pi) - T_3(\frac{5}{4}\pi)| \approx 5.46 < |f(\frac{5}{4}\pi) - T_5(\frac{5}{4}\pi)| \approx 7.88$. So, $T_3(x)$ better approximates the function $f(x) = \sin x$ at the point $x = \frac{5}{4}\pi$ than $T_5(x)$. Generally, Taylor's polynomial of higher order better approximates the function than Taylor's polynomial of lower order only locally in some neighbourhood of the point $x_0$.

## 2 Theorem of the best local polynomial approximation

This theorem and corollaries from it are inspired by theorem of the best local approximation presented in [5].

Let $P(x) = p_0 + p_1(x - x_0) + p_2(x - x_0)^2 + \ldots + p_m(x - x_0)^m$ and $Q(x) = q_0 +$

$q_1(x-x_0)+q_2(x-x_0)^2+\ldots+q_k(x-x_0)^k$ are different polynomials. Let $r$ be the smallest nonnegative integer among numbers $i=0,1,2,\ldots$ which satisfy $p_i \neq q_i$ (if $m>k$ then we put $q_{k+1}=\ldots=q_m=0$, if $m<k$ then we put $p_{m+1}=\ldots=p_k=0$).

Assume that function $f(x)$ has finite derivative of $n$ order at point $x_0$ and assume $r \leq n$.

**Theorem.** *If $p_i=\frac{f^{(i)}(x_0)}{i!}$ for all $i<r$ and $|\frac{f^{(r)}(x_0)}{r!}-p_r|<|\frac{f^{(r)}(x_0)}{r!}-q_r|$ then there exists such neighbourhood $S$ of point $x_0$ such that $\bigwedge_{x\in S}|f(x)-P(x)|<|f(x)-Q(x)|$.*

*Proof.* By Taylor's theorem we have: $f(x)-T_n(x)=(x-x_0)^n\omega(x)$, where $\omega(x)$ is a function continuous at $x_0$ and $\omega(x_0)=0$. Thus:

$$\begin{aligned}
&|f(x)-P(x)|\\
&=\Big|\left(\frac{f^r(x_0)}{r!}-p_r\right)(x-x_0)^r+\frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)^{r+1}+\cdots+\frac{f^n(x_0)}{n!}(x-x_0)^n\\
&\qquad+(x-x_0)^n\omega(x)-p_{r+1}(x-x_0)^{r+1}-\cdots-p_m(x-x_0)^m\Big|,
\end{aligned}$$

$$\begin{aligned}
&|f(x)-Q(x)|\\
&=\Big|\left(\frac{f^r(x_0)}{r!}-q_r\right)(x-x_0)^r+\frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)^{r+1}+\cdots+\frac{f^n(x_0)}{n!}(x-x_0)^n\\
&\qquad+(x-x_0)^n\omega(x)-q_{r+1}(x-x_0)^{r+1}-\cdots-q_k(x-x_0)^k\Big|.
\end{aligned}$$

Taking the factor $(x-x_0)^r$ out we have:

$$\begin{aligned}
&|f(x)-P(x)|\\
&=|(x-x_0)^r|\cdot\Big|\left(\frac{f^r(x_0)}{r!}-p_r\right)+\frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)+\cdots+\frac{f^n(x_0)}{n!}(x-x_0)^{n-r}\\
&\qquad+(x-x_0)^{n-r}\omega(x)-p_{r+1}(x-x_0)-\cdots-p_m(x-x_0)^{m-r}\Big|.
\end{aligned}$$

$$\begin{aligned}
&|f(x)-Q(x)|\\
&=|(x-x_0)^r|\cdot\Big|\left(\frac{f^r(x_0)}{r!}-q_r\right)+\frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)+\cdots+\frac{f^n(x_0)}{n!}(x-x_0)^{n-r}\\
&\qquad+(x-x_0)^{n-r}\omega(x)-q_{r+1}(x-x_0)-\cdots-q_k(x-x_0)^{k-r}\Big|.
\end{aligned}$$

The above equalities are true if $m > r$ and $k > r$. If $m \leq r$, then defining $p_{m+1} = p_{m+2} = \ldots = 0$ we have:

$$
\begin{aligned}
&|f(x) - P(x)| \\
&\quad = \Big| \left( \frac{f^r(x_0)}{r!} - p_r \right) (x-x_0)^r + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)^{r+1} + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^n \\
&\qquad\qquad + (x-x_0)^n \omega(x) \Big| \\
&\quad = |(x-x_0)^r| \cdot \Big| \left( \frac{f^r(x_0)}{r!} - p_r \right) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} \\
&\qquad\qquad + (x-x_0)^{n-r} \omega(x) \Big|
\end{aligned}
$$

and if $k \leq r$ then defining $q_{k+1} = q_{k+2} = \ldots = 0$ we have:

$$
\begin{aligned}
&|f(x) - Q(x)| \\
&\quad = \Big| \left( \frac{f^r(x_0)}{r!} - q_r \right) (x-x_0)^r + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0)^{r+1} + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^n \\
&\qquad\qquad + (x-x_0)^n \omega(x) \Big| \\
&\quad = |(x-x_0)^r| \cdot \Big| \left( \frac{f^r(x_0)}{r!} - q_r \right) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} \\
&\qquad\qquad + (x-x_0)^{n-r} \omega(x) \Big|
\end{aligned}
$$

(both numbers $k$ and $m$ cannot be at the same time less than $r$).
As $x$ approaches to $x_0$ we obtain:

$$
\begin{aligned}
\lim_{x \to x_0} \Big| &\left( \frac{f^r(x_0)}{r!} - p_r \right) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} \\
&+ (x-x_0)^{n-r} \omega(x) - p_{r+1}(x-x_0) - \cdots - p_m (x-x_0)^{m-r} \Big| \\
&= \left| \frac{f^r(x_0)}{r!} - p_r \right|,
\end{aligned}
$$

$$
\begin{aligned}
\lim_{x \to x_0} \Big| &\left( \frac{f^r(x_0)}{r!} - q_r \right) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} \\
&+ (x-x_0)^{n-r} \omega(x) - q_{r+1}(x-x_0) - \cdots - q_k (x-x_0)^{k-r} \Big| \\
&= \left| \frac{f^r(x_0)}{r!} - q_r \right|.
\end{aligned}
$$

Because of our assumption $\left|\frac{f^{(r)}(x_0)}{r!} - p_r\right| < \left|\frac{f^{(r)}(x_0)}{r!} - q_r\right|$ and the last two limits we conclude that there exists such neighbourhood $S$ of point $x_0$ such that

$$\bigwedge_{x\in S} \Big|\Big(\frac{f^r(x_0)}{r!} - p_r\Big) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} + (x-x_0)^{n-r}\omega(x) - p_{r+1}(x-x_0) - \cdots - p_m(x-x_0)^{m-r}\Big| < \Big|\Big(\frac{f^r(x_0)}{r!} - q_r\Big) + \frac{f^{r+1}(x_0)}{(r+1)!}(x-x_0) + \cdots + \frac{f^n(x_0)}{n!}(x-x_0)^{n-r} + (x-x_0)^{n-r}\omega(x) - q_{r+1}(x-x_0) - \cdots - q_k(x-x_0)^{k-r}\Big|$$

Multiplying both sides of the last inequality by $|(x-x_0)^r|$ we obtain that $\bigwedge_{x\in S} |f(x) - P(x)| < |f(x) - Q(x)|$.
In cases $m \le r$ or $k \le r$ the proof is analogous. □

**Corollary 1.** *Let $Q(x)$ be a polynomial which satisfies: there exists $i$ ($i \le n$) such that $q_i \neq \dfrac{f^{(i)}(x_0)}{i!}$ (if $m < n$ then we define $q_{m+1} = q_{m+2} = \ldots = q_n = 0$). Then there exists such neighbourhood $S$ of point $x_0$ such that, $\bigwedge_{x\in S} |f(x) - T_n(x)| < |f(x) - Q(x)|$. Particularly $Q(x)$ can be any polynomial of order not greater than $n$ different than $T_n(x)$.*

**Corollary 2.** *There exists such neighbourhood $S$ of point $x_0$ such that,* $\bigwedge_{x\in S} |f(x) - T_n(x)| \le |f(x) - T_{n-1}(x)| \le \ldots \le |f(x) - T_1(x)|$, *where every inequality from the last sequence of inequalities becomes equality if and only if when the two consecutive Taylor's polynomial of $f(x)$ in both sides of the inequality are identical.*

# 3 Visualization of the best locally approximation by Taylor's polynomials with Mathematica

Let us visualize Corollary 1 and 2 for reduced right neighbourhood $(0, 0.01)$ of the point $x_0 = 0$ using Wolfram Mathematica System [6, 7].

**Example 1.** For the Corollary 1 we define: $f(x) = e^x$, $x_0 = 0$, $T_2(x) = 1 + x + \frac{1}{2!}x^2$ and $P(x) = 1 + x - \frac{1}{2!}x^2$ for $x \in (0, 0.01)$. By Taylor's theorem we get:
$e^x - T_2(x) = e^x - (1 + x + \dfrac{1}{2!}x^2) = \dfrac{1}{3!}(e^{\tilde{x}})x^3 > 0$

and $e^x - P(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}(e^{\tilde{x}})x^3 - (1 + x - \frac{1}{2}x^2) = x^2 + \frac{1}{3!}(e^{\tilde{x}})x^3 > 0$ for $\tilde{x} \in (0,x), x \in (0,0.01)$.
Hence, we have: $|f(x) - T_2(x)| - |f(x) - P(x)| = e^x - (1 + x + \frac{1}{2}x^2) - e^x + 1 + x - \frac{1}{2}x^2 = -x^2 < 0$
and finally $\bigwedge_{x \in (0,0.001)} |f(x) - T_2(x)| < |f(x) - P(x)|$.

Let us visualize this inequality by creating a table of numerical values for both sides of inequality with step 0.001.

| $x$ | $f(x)$ | $T_2(x)$ | $P(x)$ | $\lvert f(x) - T_2(x)\rvert$ | $\lvert f(x) - P(x)\rvert$ |
|---|---|---|---|---|---|
| 0. | 1. | 1. | 1. | 0 | 0 |
| 0.001 | 1.001 | 1.001 | 1.001 | $1.667083416680558 \times 10^{-10}$ | $1.000166708341668 \times 10^{-6}$ |
| 0.002 | 1.002 | 1.002 | 1.002 | $1.334000266755581 \times 10^{-9}$ | $4.001334000266756 \times 10^{-6}$ |
| 0.003 | 1.003 | 1.003 | 1.003 | $4.503377026012934 \times 10^{-9}$ | $9.004503377026013 \times 10^{-6}$ |
| 0.004 | 1.00401 | 1.00401 | 1.00399 | $1.067734187235881 \times 10^{-8}$ | 0.00001601067734187236 |
| 0.005 | 1.00501 | 1.00501 | 1.00499 | $2.085940106338357 \times 10^{-8}$ | 0.00002502085940106338 |
| 0.006 | 1.00602 | 1.00602 | 1.00598 | $3.605406486485558 \times 10^{-8}$ | 0.00003603605406486486 |
| 0.007 | 1.00702 | 1.00702 | 1.00698 | $5.726684855523160 \times 10^{-8}$ | 0.00004905726684855523 |
| 0.008 | 1.00803 | 1.00803 | 1.00797 | $8.550427343117207 \times 10^{-8}$ | 0.00006408550427343117 |
| 0.009 | 1.00904 | 1.00904 | 1.00896 | $1.217738678140626 \times 10^{-7}$ | 0.00008112177386781406 |
| 0.01 | 1.01005 | 1.01005 | 1.00995 | $1.670841680575422 \times 10^{-7}$ | 0.0001001670841680575 |

Table 1: the values of $f(x)$, $T_2(x), P(x)$, $|f(x) - T_2(x)|$ and $|f(x) - P(x)|$ with step 0.001

We see in Table 1 that for all considered points inequality is true. Based on the Table1 we can prepare Figure 5 using logarithmic scale. Increasing WorkingPrecision and Accuracy in Mathematica Plot function we can get the continous graphs presented in Figure 6

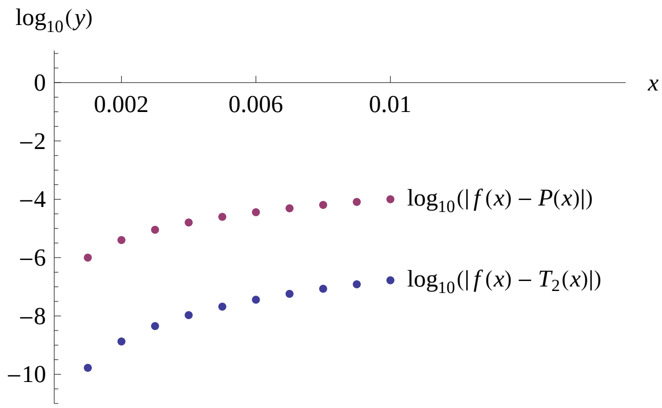


Figure 5: discrete graphs of $|f(x)-T_2(x)|$ and $|f(x)-P(x)|$ in reduced right neighbourhood $(0,0.01)$ of the point $x_0=0$ with logarithmic scale using Mathematica Plot function.

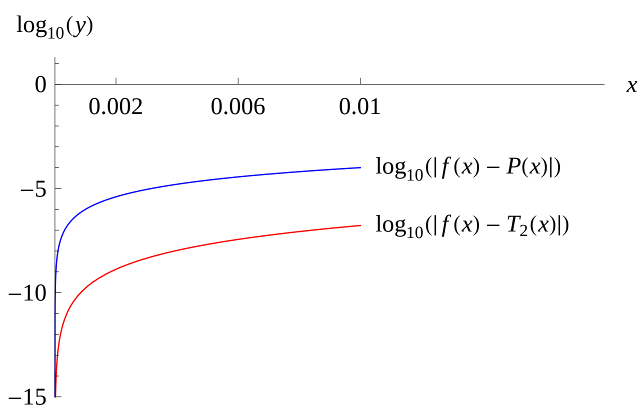


Figure 6: continuous graphs of $|f(x)-T_2(x)|$ and $|f(x)-P(x)|$ in reduced right neighbourhood $(0,0.01)$ of the point $x_0=0$ with logarithmic scale using Mathematica Plot function.

In Figure 5 we see that the graphs of $|f(x)-T_2(x)|$ and $|f(x)-P(x)|$ are separated and that $|f(x)-T_2(x)|<|f(x)-P(x)|$ for $x\in(0,0.01)$.

**Example 2.** For the Corollary 2 we define: $f(x)=\sin x, x_0=0$,
$T_3(x)=x-\frac{1}{3!}x^3$,
$T_7(x)=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7$,

$T_{11}(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \frac{1}{11!}x^{11}$.
By Taylor's theorem, for all $x \in (0, 0.01)$ we have:
$f(x) - T_3(x) = (\frac{1}{4!}\sin\tilde{x})x^4 > 0$,
$f(x) - T_7(x) = (\frac{1}{8!}\sin\tilde{\tilde{x}})x^8 > 0$,
$f(x) - T_{11}(x) = (\dfrac{1}{12!}\sin\tilde{\tilde{\tilde{x}}})x^{12} > 0$,
where $\tilde{x}, \tilde{\tilde{x}}, \tilde{\tilde{\tilde{x}}} \in (0, x)$.
Hence, for all $x \in (0, 0.01)$ we get:

$$\begin{aligned}|f(x) - T_3(x)| - |f(x) - T_7(x)| &= f(x) - T_3(x) - f(x) + T_7(x) = \frac{1}{5!}x^5 - \frac{1}{7!}x^7 \\ &= \frac{1}{7!}x^5(42 - x^2) = \frac{1}{7!}x^5(\sqrt{42} - x)(\sqrt{42} + x) > 0,\end{aligned}$$

$$\begin{aligned}|f(x) - T_7(x)| - |f(x) - T_{11}(x)| &= f(x) - T_7(x) - f(x) + T_{11}(x) = \frac{1}{9!}x^9 - \frac{1}{11!}x^{11} \\ &= \frac{1}{11!}x^9(110 - x^2) = \frac{1}{11!}x^9(\sqrt{110} - x)(\sqrt{110} + x) > 0.\end{aligned}$$

So, finally $\bigwedge_{x \in (0, 0.01)} |f(x) - T_3(x)| > |f(x) - T_7(x)| > |f(x) - T_{11}(x)|$.

Let us visualize this double inequality by create a table of values for all sides of inequality with step 0.001.

| $x$ | $f(x)$ | $T_3(x)$ | $T_7(x)$ | $T_{11}(x)$ | $\lvert f(x) - T_3(x)\rvert$ | $\lvert f(x) - T_7(x)\rvert$ | $\lvert f(x) - T_{11}(x)\rvert$ |
|---|---|---|---|---|---|---|---|
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | $8.45678 \times 10^{-18}$ | $2.75573 \times 10^{-33}$ | $1.60590 \times 10^{-49}$ |
| 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | $2.66714 \times 10^{-16}$ | $1.41093 \times 10^{-30}$ | $1.31556 \times 10^{-45}$ |
| 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | $2.02529 \times 10^{-15}$ | $5.42411 \times 10^{-29}$ | $2.56033 \times 10^{-43}$ |
| 0.004 | 0.00399999 | 0.00399999 | 0.00399999 | 0.00399999 | $8.53397 \times 10^{-15}$ | $7.22398 \times 10^{-28}$ | $1.07770 \times 10^{-41}$ |
| 0.005 | 0.00499998 | 0.00499998 | 0.00499998 | 0.00499998 | $2.60417 \times 10^{-14}$ | $5.38229 \times 10^{-27}$ | $1.96033 \times 10^{-40}$ |
| 0.006 | 0.00599996 | 0.00599996 | 0.00599996 | 0.00599996 | $6.47997 \times 10^{-14}$ | $2.77714 \times 10^{-26}$ | $2.09742 \times 10^{-39}$ |
| 0.007 | 0.00699994 | 0.00699994 | 0.00699994 | 0.00699994 | $1.40058 \times 10^{-13}$ | $1.11204 \times 10^{-25}$ | $1.55594 \times 10^{-38}$ |
| 0.008 | 0.00799991 | 0.00799991 | 0.00799991 | 0.00799991 | $2.73066 \times 10^{-13}$ | $3.69868 \times 10^{-25}$ | $8.82855 \times 10^{-38}$ |
| 0.009 | 0.00899988 | 0.00899988 | 0.00899988 | 0.00899988 | $4.92073 \times 10^{-13}$ | $1.06763 \times 10^{-24}$ | $4.08199 \times 10^{-37}$ |
| 0.01 | 0.00999983 | 0.00999983 | 0.00999983 | 0.00999983 | $8.33332 \times 10^{-13}$ | $2.75573 \times 10^{-24}$ | $1.60590 \times 10^{-36}$ |

Table 2: the values of $f(x), T_3(x), T_7(x), T_{11}(x), |f(x) - T_3(x)|, |f(x) - T_7(x)|$ and $|f(x) - T_{11}(x)|$ with step 0.001

We see in Table 2 that for all considered points double inequality is true. Based on the Table 2 we can prepare Figure 7 using logarithmic scale.

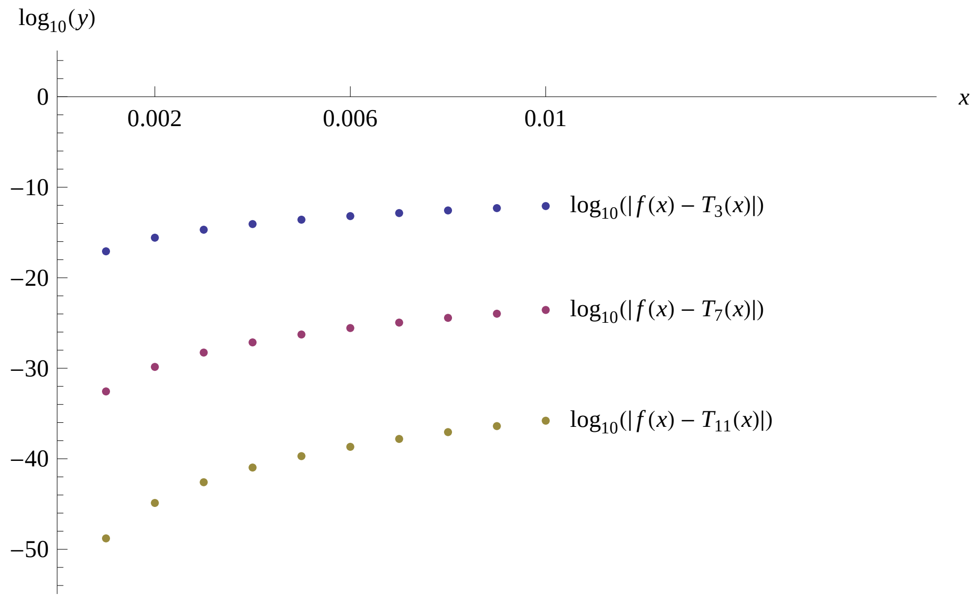


Figure 7: discrete graphs of $|f(x) - T_3(x)|, |f(x) - T_7(x)|$ and $|f(x) - T_{11}(x)|$ in reduced right neighbourhood $(0, 0.01)$ of the point $x_0 = 0$ with logarithmic scale using Mathematica ListPlot function.

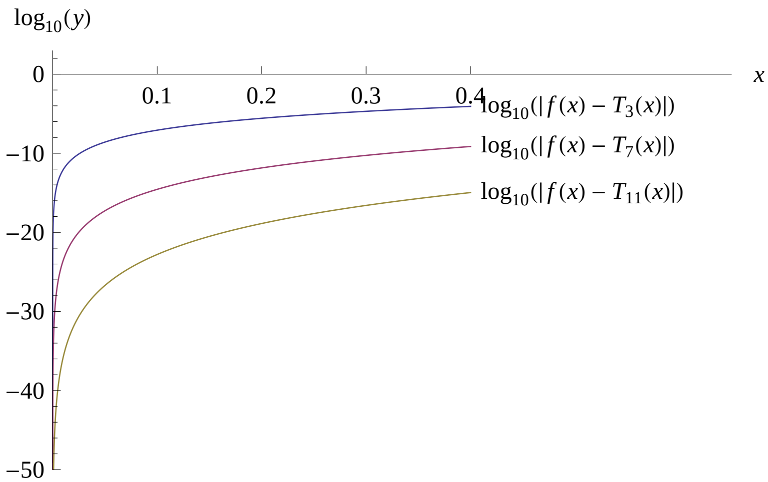


Figure 8: continous graphs of $|f(x)-T_3(x)|, |f(x)-T_7(x)|$ and $|f(x)-T_{11}(x)|$ in reduced right neighbourhood $(0, 0.4)$ of the point $x_0 = 0$ with logarithmic scale using Mathematica Plot function.

In Figures 6 and 7 we see that graphs of $|f(x)-T_3(x)|, |f(x)-T_7(x)|$ and $|f(x)-T_{11}(x)|$ are separated.

## Summary


In this paper the authors discuss graphic presentation of Taylor's polynomials in context of local approximation of a function. In popular way of graphic presentation Taylor's polynomials, graph of the function $f(x)$ and graphs of its Taylor's polynomials seem to overlap in a neighbourhood of the point $x = x_0$. Using logarithmic scale to present graphs we can separate graphs of differences between function and its Taylor's polynomials. To prepare graphs Mathematica System was used.


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