The root lattice A_2 in the construction of tilings and algebraic hypersurfaces with many singularities

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In [7] we have shown that special types of simplicial arrangements of d lines contain simple arrangements which are related to a class of bivariate polynomials $J_d(x,y)$ having many critical points with few critical values. The polynomials have been used in the construction of algebraic surfaces with many A and D singularities [4, 5, 6, 7].

Tilings exhibiting non crystallographic symmetries have been significant in the past decades in the field of quasicrystals. The root lattice A_4 was considered in [1] to generate planar tilings with five-fold symmetry by projection methods. Certain pseudoline configurations inside the fundamental region of the affine Weyl group associated to the root lattice A_2 can be transformed into simple arrangements of lines containing the triangular prototiles of substitution tilings with n-fold symmetry. The analysis of the critical points of $J_d(x,y)$ allows us to define other sets of pseudolines in the fundamental region which are transformed into the simplicial arrangements containing the inflated prototiles [2]. Topological invariants of tiling spaces connected with the simplicial arrangements have been studied in ([3] and references within), where we have shown that there are five-fold and nine-fold symmetry tiling spaces having minimal first cohomology groups, a property that distinguish them from others with the same symmetries. Random tilings can be generated from both the line and the pseudoline configurations [2].

On the other hand, by following Hirzebruch's methods [9, 10] applied to special line configurations, threefolds with trivial canonical bundle and absolute value of the Euler number not large but different from zero can be obtained. Mathematica [11] and Singular [8] computer algebra systems are used.

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