Symbolic-Numeric Computing: A Polynomial System Arising in Image Analysis of Point Cloud Data

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Systems of polynomial equations with parameters arise in many fields, such as geometric computing, flexibility of molecules, chemical reactions, game theory, image analysis, operations research, and differential equations. In most applied problems, the best method for their symbolic solution is the Dixon-EDF resultant [**?**]. We will briefly describe the method itself, then discuss a problem arising from surface reconstruction from point cloud data. We focus on our recent work with B. Palancz and J. Awange on image analysis. Given a point cloud created by a laser scan (LIDAR) [**?**], we want to discern underlying shapes. This entails separating “inliers" from “outliers" by the maximization of the likelihood function of a dual Gaussian mixture distribution. This is fine example of a symbolic-numeric method.

We introduce a new robust technique employing expectation maximization to separate outliers (extraneous data points) from inliers (true data points) iteratively, represented by different Gaussian distributions. Since in every iteration step, a new parameter estimation should be carried out, a key point is to solve this parameter estimation as fast as possible. To do that, the problem of numerical global maximization of the likelihood function of the Gaussian mixture was transformed into the solution of a multivariate polynomial system. The symbolic solution of the resulting polynomial system consisting of four equations is quite challenging because of the high number of parameters. In order to solve it, a linear transformation was required to reduce the total degrees of the polynomials. This reduced system was solved successfully via Dixon-EDF resultant method.

Some of this was reported on at ACA 2014 [**?**]. However, there is now a significant new numerical technique used in the iterative step. We compare our results with other robust methods such as Danish and Random Sample Consensus methods on the data set of a real laser scanning experiment. We find that our method is much faster and more robust.

References

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