How to Use CAS (Maple) to Help Students Learn Number Theory

M. Durcheva[1]

[1] *Technical University of Sofia, Bulgaria, {mdurcheva66}@gmail.com*

At the Technical University of Sofia (TUS), some topics of Number Theory are taught in the "Discrete mathematics" course. The notions of the Number Theory are very important for the students of computer sciences, as they are necessary, for instance, for the "Cryptography" course.

As it is pointed out in [1], for an effective mathematical education it is useful the teacher to:

- challenge his students to think deeply about the problems they are solving;
- influence learning by posing challenging and interesting questions;
- encourage students' ability to "do" mathematics.

This paper will highlight some successful strategies for enhancing students' learning that the author has used in teaching topics of Number Theory at the TUS. In particular, the use of the CAS (Maple) expands students' ability to "do" mathematics and to reach beyond the solutions and algorithms required to solve the problem.

There are some problems in Number Theory where we can apply CAS in proving a statement, as it is in the

Problem 1. Prove that for all prime p, the number $p^{2017} - 1$ is a composite.

It is clear that for all odd primes the proposition is obviously true. It remains, students using CAS, to check it for $p = 2$.

Another approach we use to influence students' learning, is to pose concrete small problems and to ask students to make a hypothesis and then to prove it.

Problem 2. a) Check whether the numbers $x = 3^{2016} + 2^{2018}$ and $y = 2017^4 + 4^{2017}$, are primes.

b) Factorize $z = 2^{4n+2} + 1$.

c) Find the general form of x, y, and z, and try to make a conclusion.

Maple gives the factorization of the general form $a^4 + 4b^4$.

Another interesting types of numbers are Fermat numbers. To introduce the properties of these numbers to the students, the teacher could include the following

Problem 3. a) Check whether the numbers of the type $2^{2^n} + 1$ are primes.

b) Find the last digit of their decimal representation for $n \geq 2$.

For 3,b, CAS helps students to make a conjecture and then most of them attempt to prove it on their own.

Because there are some theorems in Number Theory, whose proofs are very complicated or rely on advanced mathematics, it is useful students "to be convinced" in their truthfulness, as in the

Prime number theorem. [2, 3] The function $\pi(x)$ ($\pi(x)$ is the number of primes $\leq x$) is asymptotic to $x/\ln(x)$, in the sense that

$$\lim_{x\to\infty} \frac{\pi(x)}{x/\ln(x)} = 1.$$

At the TUS, Number Theory is taught in the first year, so we do not include the proof of the this theorem in our course. That is way, it is very important for us to motivate students to observe the truthfulness of the Prime number theorem. Maple can help the teacher significantly in this direction. The teacher may pose the following

Problem 4. Graph on the same plot the graphs of the functions $\pi(x)$ and $x/\ln(x)$. Make a conclusion.

Another approach to enhance students' learning is to pose them challenging questions that not only stimulate students' innate curiosity, but also encourages them to investigate further. For instance, when studying numerical function $\tau(n)$, the teacher can ask the following

Problem 5. Study the function $\tau(n)$ (using Maple). What conjectures can you make about it? Is there a formula for $\tau(n)$? Is the function $\tau(n)$ multiplicative?

One of the most difficult topics in Number theory are **Diophant equations**. However, using Maple, some kind of linear Diophant equations can be easily solved. Here the teacher could pose intriguing problems to motivate students to study this topic.

While we are not in a position to run a controlled experiment to prove the efficiency of these teaching methods, there have been several benefits in our classroom. The students are more engaged, more likely to try to solve the problems on their own, and at the end, they score higher on examinations.

References

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