Computer-Algebra-Aided Chebyshev Methods for Ordinary Differential Equations

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The solution of ordinary differential equation can be approximated by a linear combination of so called basis functions. Using the Chebyshev Polynomials as the basis functions, the approximation can be expressed as

$$y(x) = \sum_{r=0}^{\infty} a_r T_r(x) \tag{1}$$

where $T_r(x)$'s are Chebyshev Polynomials of degree r, and a_r's are the coefficients to be determined. In practice, we seek the approximation using a truncated expression of (1), namely,

$$\sum_{r=0}^{n} a_r T_r(x) \tag{2}$$

An online Computer Algebra System (CAS) is used to generate and subsequently solve a system of equations concerning a finite number of a_r's. The use of CAS allows the retention of more a_r's in (2). It also obviates the need for the traditional pad and pencil computations.

Examples will be given to illustrate this approach in solving initial value problems, boundary value problems as well as eigenvalue problems for ordinary differential equations whose coefficients and other terms are themselves polynomials.

References

[1] L. Fox and D.F. Mayers, *Numerical Solution of Ordinary Differential Equations*, pp. 179-197 (1987).

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[3] *Omega: A Computer Algebra System Explorer*, at http://www.omega-math.com