

# Generating Power Summation Formulas Using a Computer Algebra System

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Mathematical induction is often used in classroom to *prove* various Power Summation Formulas such as

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad (1)$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad (2)$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4} \quad (3)$$

However, how the formulas are obtained in the first place is rarely discussed.

In this presentation, we will *construct* the Power Summation Formulas. Specifically, a recursive algorithm is derived and its implementation in Computer Algebra generates the formulas. A closer look at this algorithm also reveals the generated formulas can also be obtained by solving an initial-value problem of difference equation symbolically.

## References

- [1] J. Gullberg, *Mathematics From the Birth of Numbers* (1997).
- [2] *Omega: A Computer Algebra System Explorer*, at <http://www.omega-math.com>