

Elements of Calculus I, MATH 180 Midterm 1
Janet Vassilev

(1) (12 points) Find limits of the following:

(a) $\lim_{x \rightarrow 5} x^2 - 3x + 2.$

$$= 5^2 - 3 \cdot 5 + 2 = 12$$

(b) $\lim_{x \rightarrow 2} \frac{2x - 4}{x^2 + 3x - 10} = \lim_{x \rightarrow 2} \frac{2(x-2)}{(x+5)(x-2)}$

$$= \lim_{x \rightarrow 2} \frac{2}{x+5} = \frac{2}{7}$$

(c) $\lim_{x \rightarrow -\infty} \frac{2x^3 + 4x - 7}{4x^2 - 9} = \lim_{x \rightarrow -\infty} \frac{(2x^3 + 4x - 7) \frac{1}{x^2}}{(4x^2 - 9) \frac{1}{x^2}}$

$$= \lim_{x \rightarrow -\infty} \frac{2x + \frac{4}{x} - \frac{7}{x^2} \rightarrow 0}{4 - \frac{9}{x^2} \rightarrow 0}$$

$$= -\infty$$

(2) (12 points) For the function $G(x) = \begin{cases} x^2 - 1 & \text{for } x < 0 \\ \frac{1}{x-1} & \text{for } 0 \leq x < 2 \\ 2x + 1 & \text{for } x \geq 2, \end{cases}$ determine the following:

(a) $\lim_{x \rightarrow 0^-} G(x).$

$$= \lim_{x \rightarrow 0^-} x^2 - 1 = 0 - 1 = -1$$

(b) $\lim_{x \rightarrow 0^+} G(x).$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x-1} = \frac{1}{0-1} = -1$$

(c) $\lim_{x \rightarrow 0} G(x).$

$$\text{Since } \lim_{x \rightarrow 0^-} G(x) = \lim_{x \rightarrow 0^+} G(x) = -1, \\ \lim_{x \rightarrow 0} G(x) = -1$$

(d) $\lim_{x \rightarrow 2^-} G(x).$

$$= \lim_{x \rightarrow 2^-} \frac{1}{x-1} = \frac{1}{2-1} = 1$$

(e) $\lim_{x \rightarrow 2^+} G(x).$

$$= \lim_{x \rightarrow 2^+} 2x + 1 = 2 \cdot 2 + 1 = 5$$

(f) $\lim_{x \rightarrow 2} G(x).$

$$\text{Since } \lim_{x \rightarrow 2^-} G(x) \neq \lim_{x \rightarrow 2^+} G(x) \\ \lim_{x \rightarrow 2} G(x) \text{ does not exist.}$$

(3) (12 points) Suppose $\lim_{x \rightarrow 4} f(x) = 2$ and $\lim_{x \rightarrow 4} g(x) = -7$. Determine:

(a) $\lim_{x \rightarrow 4} f(x) - g(x) = 2 - (-7) = 9$

(b) $\lim_{x \rightarrow 4} \frac{3f(x)}{1 + g(x)} = \frac{3 \cdot 2}{1 + (-7)} = \frac{6}{-6} = -1$

(c) $\lim_{x \rightarrow 4} f(x)g(x) = 2(-7) = -14$

(4) (10 points) For the function $f(x) = \begin{cases} 2x^2 - x & \text{for } x < -1 \\ 3x + k & \text{for } x \geq -1, \end{cases}$ determine the value of k which will make $f(x)$ continuous at $x = -1$.

To be continuous

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) = f(-1) = 3(-1) + k = -3 + k$$

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} 2x^2 - x = 2(-1)^2 - (-1) = 3$$

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} 3x + k = -3 + k$$

$$\text{so } 3 = -3 + k$$

$\therefore k = 6$ will make $f(x)$ continuous

- (5) (10 points) Using the four step process or the definition of the derivative find the derivative of $f(x) = \frac{1}{x}$.

$$\begin{aligned}
 \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \\
 &= \lim_{h \rightarrow 0} -\frac{1}{x(x+h)} = -\frac{1}{x^2}
 \end{aligned}$$

- (6) (10 points) Find the equation of the tangent line to $f(x) = 5\sqrt{x} - 2x + 3$ at the point $(4, 5)$.

$$f'(x) = \frac{5}{2}x^{-\frac{1}{2}} - 2$$

$$f'(4) = \frac{5}{2} \cdot 4^{-\frac{1}{2}} - 2 = -\frac{3}{4} \text{ is}$$

the slope of the tangent line at $(4, 5)$.

The line is

$$y - 5 = -\frac{3}{4}(x - 4) \text{ or}$$

$$y = -\frac{3}{4}x + 8$$

(7) (16 points) Find derivatives of the following functions:

(a) $f(x) = \frac{x^5 - 3x^2 + 7}{x} = x^4 - 3x + 7x^{-1}$

$$f'(x) = 4x^3 - 3 - 7x^{-2}$$

(b) $f(x) = (2x^4 - 8x^3 + 12)^{10}$

$$f'(x) = 10(2x^4 - 8x^3 + 12)^9 (8x^3 - 24x^2)$$

(c) $g(y) = y^3(2y - 5)^7$

$$g'(y) = y^3 \cdot 7(2y - 5)^6 \cdot 2 + 3y^2(2y - 5)^7$$

(d) $h(t) = \frac{2t}{t^2 - 1}$

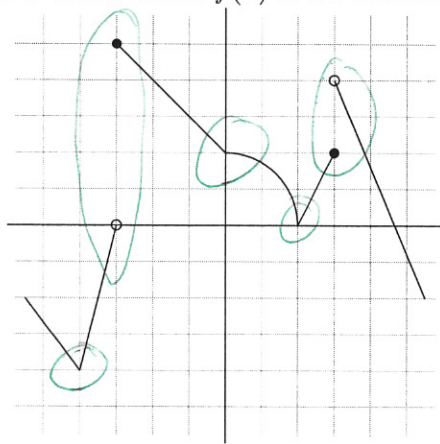
$$h'(t) = \frac{(t^2 - 1) \cdot 2 - 2t(2t)}{(t^2 - 1)^2}$$

- (8) (10 points) If $f(x) = g(x)h(x)$, $F(x) = \frac{g(x)}{h(x)}$ and $g(5) = -0.5$, $h(5) = 13$, $g'(5) = 10$ and $h'(5) = -1$ determine $f'(5)$ and $F'(5)$.

$$f'(5) = g(5)h'(5) + g'(5)h(5) \\ = (-0.5)(-1) + 10 \cdot 13 = 130.5$$

$$F'(5) = \frac{13 \cdot 10 - (-0.5)(-1)}{(13)^2} \\ = \frac{129.5}{169} = \frac{259}{338}$$

- (9) (8 points) Looking at the graph of $f(x)$ below where each grid division is one unit, determine the x -values where $f(x)$ is not differentiable.



The graph
is not differentiable
at
 $x = -4$, $x = -3$, $x = 0$,
 $x = 2$ & $x = 3$.