Elements of Calculus I, MATH 180 Midterm 2
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(1) The weekly demand for the Peach Tablet is $p = 1000 - 0.1x$ for $0 \le x \le 10,000$. Where p is the wholesale unit price and x is the quantity demanded.
  (a) Find both the revenue function and the marginal revenue function.
  (b) Determine $R'(3000)$ and describe what this quantity says about the Revenue acquired in producing the 3001st Tablet.

a) $R(x) = x \cdot p = x(1000 - .1x)$
$= 1000x - .1x^2$

$R'(x) = 1000 - .2x$

b) $R'(3000) = 1000 - .2(3000)$
$= 1000 - 600 = 400$

The marginal revenue computed at 3000 predicts the revenue brought in by producing the 3001st product once the 3000th has been produced. So the approximate revenue of 3001st product is \$400.

(2) Find the first three derivatives of $f(x) = 8\sqrt{x} - \dfrac{5}{x^2}$

$f'(x) = 8 \cdot \frac{1}{2} x^{-1/2} + 10x^{-3}$
$= 4x^{-1/2} + 10x^{-3}$

$f''(x) = -2x^{-3/2} - 30x^{-4}$

$f'''(x) = 3x^{-5/2} + 120x^{-5}$

(3) Find an equation for the tangent line to $(2x - y - 1)^3 = 8x$ at the point $(1, -1)$.

$$3(2x-y-1)^2\left(2-\frac{dy}{dx}\right) = 8$$

$$2 - \frac{dy}{dx} = \frac{8}{3(2x-y-1)^2}$$

$$\frac{dy}{dx} = 2 - \frac{8}{3(2x-y-1)^2}$$

$$m = 2 - \frac{8}{3(2+1-1)^2} = 2 - \frac{8}{12} = 2 - \frac{2}{3} = \frac{4}{3}$$

$$y+1 = \frac{4}{3}(x-1)$$

or $y = 4/3\,x - \frac{7}{3}$

(4) Leah is blowing air into a spherical soap bubble at the rate of $5\text{cm}^3/\text{sec}$. Find the rate the radius is changing when the radius is 3 centimeters. Recall that the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4 \cdot \pi \cdot 3^2 \frac{dr}{dt}$$

$$\frac{5}{36\pi} \text{ cm/sec} = \frac{dr}{dt}$$

(5) Estimate $\sqrt{24}$ using differentials.

25 is a perfect square

so $dx = 24 - 25 = -1$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2} x^{-1/2}$

$$\sqrt{24} \approx \sqrt{25} + f'(25)\,dx$$

$$\approx \sqrt{25} + \frac{1}{2}(25)^{-1/2}(-1)$$

$$\approx 5 - \frac{1}{10} = 4.9$$

(6) Find the differential of $y = x(x^3 + 3x + 1)^5$.

$$dy = \left[x \cdot 5(x^3+3x+1)^4(3x^2+3) + (x^3+3x+1)^5\right] dy$$

(7) Determine the intervals where $f(x) = \dfrac{x^2}{x^2 - 9}$ is increasing or decreasing.

$$f'(x) = \frac{(x^2-9)2x - x^2(2x)}{(x^2-9)^2}$$

$$= \frac{-18x}{(x^2-9)^2} = 0 \Rightarrow x = 0$$

always positive if $x \neq \pm 3$

$++ \; -3 \; ++ \; 0 \; -- \; 3 \; --$ $f'(x)$ [use $-18x$]

increasing $(-\infty,-3), (-3,0)$

decreasing $(0,3), (3,\infty)$

(8) Consider $f(x) = \frac{1}{2}x^2 - 3\sqrt[3]{x^4}$.

(a) Identify the x-values if there are any where $f(x)$ has a relative maximum.

(b) Identify the x-values if there are any where $f(x)$ has a relative minimum.

$$f'(x) = x - 4x^{1/3} = x^{1/3}(x^{2/3} - 4)$$

$x^{1/3} = 0$ & $x^{2/3} - 4 = 0$

$$\Rightarrow x^{2/3} = 4$$

$$x = \pm 4^{3/2} = \pm 8$$

$- \; -8 \; - \; - \; 0 \; + \; + \; 8 \; +$ $x^{1/3}$

$- \; -8 \; - \; 0 \; - \; - \; 8 \; + \; +$ $x^{1/3} - 2$

$- \; - \; -8 \; + \; + \; 0 \; + \; + \; 8 \; +$ $x^{1/3} + 2$

$- \; - \; -8 \; + \; + \; + \; 0 \; - \; - \; - \; 8 \; ++$ $f'(x)$

local min at $x = -8$ & $x = 8$

local max at $x = 0$

or $f''(x) = 1 - 4/3\, x^{-2/3}$ $\quad f''(0) =$ undefined cant use

$f''(-8) = f''(8) = 1 - \frac{1}{3} = 2/3$ so rel min

(9) Determine the intervals where $f(x) = x^4 - 4x^3 - 48x^2$ is concave up or concave down.

$$f'(x) = 4x^3 - 12x^2 - 96x$$

$$f''(x) = 12x^2 - 24x - 96 = 0$$

$$= 12(x^2 - 2x - 8)$$

$$= 12(x-4)(x+2)$$

$$x = -2 \text{ or } x = 4$$

	-2		4		
— —		— —		+ +	$x-4$
— — —		+ +		+ +	$x+2$
+ +		— — —		+ +	$f''(x)$

concave up: $(-\infty, -2)$ & $(4, \infty)$

concave down: $(-2, 4)$.

(10) Suppose f is a function which satisfies the following $f'(4) = 0$ and $f''(4) = 7$ and $f'(8)$ is undefined but $f(8) = 2$.

(a) What can you say about the point $(4, f(4))$?

(b) Can you say anything about the point $(8, 2)$?

a) By the 2nd derivative test $(4, f(4))$ is a relative minimum.

b) Since $f'(8)$ is undefined $(8, 2)$ is a critical point. We can't say if it is a relative min, relative max or neither.