Key 4pts each

**Elements of Calculus I, MATH 180 Quiz 12**

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(1) Evaluate the following:

(a) $\int e^{2x} - e^{-2x} dx$ $= \int e^{2x}dx - \int e^{-2x}dx = \frac{1}{2}\int e^{2x}2dx + \frac{1}{2}\int e^{-2x}(-2dx)$

$u = 2x$, $du = 2dx$ $\quad$ $v = -2x$, $v = -2dx$

$= \frac{1}{2}\int e^u du + \frac{1}{2}\int e^v dv)$

$= \frac{1}{2}e^u + \frac{1}{2}e^v + C$

$= \boxed{\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + C}$

(b) $\int x(x-3)^4 dx$ $= \int (u+3)u^4 du$

$u = x-3 \quad x = u+3$, $du = dx$

$= \int u^5 + 3u^4 du$

$= \frac{1}{6}u^6 + \frac{3}{5}u^5 + C$

$= \boxed{\frac{1}{6}(x-3)^6 + \frac{3}{5}(x-3)^5 + C}$

(2) Suppose $f'(x) = x(x^2+1)^{-1} + 2x$ and $f(0) = 2$, find $f(x)$.

$f(x) = \int x(x^2+1)^{-1} + 2x\, dx$

$= \frac{1}{2}\int 2x(x^2+1)^{-1}dx + \int 2x dx$

$= \frac{1}{2}\int u^{-1}du + \int 2x dx$

$= \frac{1}{2}\ln|u| + x^2 + C = \frac{1}{2}\ln|x^2+1| + x^2 + C$

$2 = \frac{1}{2}\ln(1) + 0 + C$

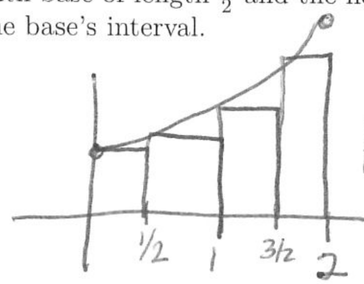
$2 = C$

so

$\boxed{f(x) = \frac{1}{2}\ln|x^2+1| + x^2 + 2}$

(3) Approximate the area under $f(x) = x^2 + 1$ on the interval $[0, 2]$ using four rectangles each with base of length $\frac{1}{2}$ and the height of each rectangle is given by the left hand endpoint of the base's interval.

full credit ✓




$A \approx \boxed{\frac{1}{2}(0^2+1) + \frac{1}{2}((\tfrac{1}{2})^2+1) + \frac{1}{2}(1^2+1) + \frac{1}{2}((\tfrac{3}{2})^2+1)}$

$= \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{5}{4} + \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot\frac{13}{4}$

$= \frac{1}{2} + \frac{5}{8} + 1 + \frac{13}{8}$

$= \frac{4+5+8+13}{8} = \frac{30}{8}$