Key

Elements of Calculus I, MATH 180 Quiz 7

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(1) Using the 2nd derivative test, determine the x-values where $f(x) = 2x^3 - 3x^2 - 36x + 60$ a) has a relative maximum and b) has a relative minimum.

5 pts

+1 $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$

+1 $f''(x) = 12x - 6$

+1 critical values at $x = 3$ & $x = -2$

+1 $f''(3) = 36 - 6 = 30 > 0$ so rel min at $x = 3$

+1 $f''(-2) = -24 - 6 = -30 < 0$ so rel max at $x = -2$.

(2) Consider $f(x) = \sqrt[3]{x^4} - 4\sqrt[3]{x}$. Identify a) the intervals where $f(x)$ is concave up, b) the intervals where $f(x)$ is concave down, and c) the x-values where $f(x)$ has a point of inflection.

5 pts

+1 $f'(x) = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$

+1 $f''(x) = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = \frac{4}{9}(x^{-5/3})(x+2)$

+1

	-2	0		
$- - -$	$- -$	$+ + + +$	$\frac{4}{9}x^{-5/3}$
$- - - -$	$+ +$	$+ + +$	$x+2$
$+ + + +$	$- - -$	$+ + +$	$f''(x)$

+1 { a) concave up $(-\infty, -2)$ & $(0, \infty)$
b) concave down $(-2, 0)$

+1 c) inflection points at $x = 0$ & $x = -2$ since 0 & 2 in domain & concavity changes

(3) Why doesn't the 2nd derivative test allow us to show there is a relative maximum at $x = 3$ for the function $f(x) = (3 - x)^3$?

2 pts extra credit

$f'(x) = 3(3-x)^2(-1)$

$f''(x) = -3 \cdot 2(3-x)(-1) = 6(3-x)$

$f''(3) = 0$ so 2nd derivative test is inconclusive.