Solutions

MATH 314–Linear Algebra
Quiz 4
Dr. Janet Vassilev

(1) Give 2 different examples of vector spaces we have talked about in class. You do not need to prove they are vector spaces.

$\mathbb{R}^n$ with componentwise addition and usual scalar multiplication $\alpha(x_1,\dots,x_n)^T = (\alpha x_1,\dots,\alpha x_n)^T$

P_n with usual addition of polynomials and usual scalar multiplication.

(2) Let $S = \{p(x) \in P_2 \mid p(0) = 1\}$. Determine if S is a subspace of P_2.

S is not a subspace since $p(x), q(x) \in S \Rightarrow p(0) = q(0) = 1$ but $p(x)+q(x)$ has $(p+q)(0) = 1+1 = 2 \neq 1$ so $p(x)+q(x) \notin S$. Since S fails closure of addition S cannot be a subspace.

(3) Do the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}2\\4\\4\end{pmatrix}$ span $\mathbb{R}^3$?

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & x \\ 0 & 1 & 4 & y \\ 0 & 1 & 4 & z \end{array}\right) \xrightarrow{-R_2+R_3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & x \\ 0 & 1 & 4 & y \\ 0 & 0 & 0 & z-y \end{array}\right)$$

is inconsistent since y is not necessarily z so the vectors do not span $\mathbb{R}^3$.

(4) Are the vectors $1+x$ and $2-x$, linearly independent?

$$c_1(1+x) + c_2(2-x) = 0 \Rightarrow \begin{array}{l} c_1 + 2c_2 = 0 \\ c_1 - c_2 = 0 \end{array}$$

Then $c_1 = c_2$ & $3c_1 = 0$

so $c_1 = c_2 = 0$

Thus $1+x$ and $2-x$ are linearly independent.

Solutions

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Quiz 5
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(1) Given a set of three vectors in $\mathbb{R}^2$, can the vectors be:

(a) linearly independent?

No $3>2$. A linearly independent set has at most 2 vectors in $\mathbb{R}^2$.

(b) a spanning set?

Yes a spanning set will have 2 or more vectors so a set of 3 could span $\mathbb{R}^2$.

(2) If four vectors are linearly independent in a vector space V, can you say anything about the dimension of V?

The dimension must be at least 4 since a basis must be linearly independent and spanning.

(3) Give a basis for the subspace $W = \{(a+b, a-b, 2a, 3b)^T \mid a, b \in \mathbb{R}\}$ of $\mathbb{R}^4$.

$\begin{pmatrix} a+b \\ a-b \\ 2a \\ 3b \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}$ is a typical element of W. Since $\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}$ are linearly independent and span W the basis is $\left\{\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}\right\}$.

(4) Let $B = \left\{\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}\right\}$ be a basis for $\mathbb{R}^2$. Find the coördinate vector of $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ with respect to the basis B.

$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ is the transition matrix from B to $\{\bar{e}_1, \bar{e}_2\}$.

So $A^{-1} = \frac{1}{9-8}\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$ is the transition matrix from $\{\bar{e}_1, \bar{e}_2\}$ to B.

So $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 15-12 \\ -10+9 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \left[\begin{pmatrix} 5 \\ 3 \end{pmatrix}\right]_B$.

Solutions.

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Quiz 6
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(1) Suppose A is a 4×7 matrix.

(a) If rank A=3, what is the dimension of the nullspace?

$7-3=4$ is the nullity.

(b) If the row space of A is generated by the row vectors $(1\ 0\ 2\ 3\ 0\ 1\ 0)$, $(0\ 0\ 0\ 0\ 1\ -1\ 0)$ and $(0\ 0\ 0\ 0\ 0\ 0\ 1)$, which columns of A generate the column space of A?

The matrix A will have a basis generated by columns 1, 5 and 7 since A row equivalent to

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Big] \text{to make right size}$$

(2) Is $L(x,y) = x + y - 1$ a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$? Justify your answer.

$$L((x,y)+(z,w)) = L(x+z, y+w) = x+z+y+w-1$$

$$L(x,y)+L(z,w) = x+y-1+z+w-1 = x+z+y+w-2$$

so not a linear trans.

(3) Is $L(x) = (x, x, x)$ a linear transformation from $\mathbb{R}$ to $\mathbb{R}^3$? Justify your answer.

$$L(x+y) = (x+y, x+y, x+y) = (x,x,x)+(y,y,y) = L(x)+L(y)$$

$$L(\alpha x) = (\alpha x, \alpha x, \alpha x) = \alpha(x,x,x) = \alpha L(x)$$

(4) Give a matrix representation A for $L\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ x-y+2z \end{pmatrix}$ with respect to the standard bases in $\mathbb{R}^3$ and $\mathbb{R}^2$.

$$A = (L(\bar{e}_1), L(\bar{e}_2), L(\bar{e}_3)) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

(5) If A and B are matrix representations for a linear operator L, what can you say about A and B?

A and B are similar.