

**COLLEGE GEOMETRY  
HOMEWORK 2**

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Due February 13 by 8 a.m.

- (1) For each of Examples 2.8 and 2.10 in the text, state the axioms of incidence geometry that do not hold.
- (2) Suppose we take the first three axioms from incidence geometry and replace the 4th by:  
**Incidence Axiom 4'** Given any line, there are at least 3 distinct points that lie on it.  
What is the smallest number of points in a model for this geometry? (Hint: think of the models that we have already seen for incidence geometry.)
- (3) Show that the following statement is independent of the axioms of incidence geometry:  
Given any line, there are at least two distinct points that do not lie on it.
- (4) Show that the following statement is independent of the axioms of incidence geometry:  
Given any point, there are at least three distinct lines that contain it.
- (5) Show that the Fano plane and the 7-point plane are not isomorphic to each other. (Hint: It is not enough just to show that a particular correspondence is not an isomorphism; you need to demonstrate that there cannot exist any isomorphism between the two models.)
- (6) Define a model of incidence geometry with points 1, 2, 3, 4, 5, 6 and lines  $\{1, 2, 3\}$ ,  $\{3, 4, 5\}$ ,  $\{5, 6, 1\}$ ,  $\{1, 4\}$ ,  $\{2, 5\}$ ,  $\{3, 6\}$ ,  $\{2, 6\}$ ,  $\{4, 6\}$ ,  $\{2, 4\}$ . Which if any of the parallel postulates is satisfied by this model? Explain your answer.
- (7) Prove the following: Given any line  $\ell$ , there exists a point that does not lie on  $\ell$ .
- (8) Prove the following: Given two distinct points  $A$  and  $B$ , there exists a line that contains  $A$  but not  $B$ .
- (9) Prove the following: If  $A, B$  and  $C$  are noncollinear points, then  $\overleftrightarrow{AB} \neq \overleftrightarrow{AC}$ .