Homework 1 – Additional Problems
for MATH 421

From our text, please do problems 2.2, 2.4, 2.5 and 2.6. Then do the following problems.

(A) Find a generator for the ideal $(85, 1 + 13i)$ in $\mathbb{Z}[i]$.

(B) Prove that $\mathbb{Z}[\sqrt{2}]$ is a Euclidean domain. (Hint: Consider $\delta(a + b\sqrt{2}) = |a^2 - 2b^2|$)

(C) Show that every field is a Euclidean domain.

(D) If a and b are associates in a Euclidean domain D with norm δ. Show that $\delta(a) = \delta(b)$.

(E) Let D be a Euclidean domain and let δ be the norm on D. Show that for nonzero $a, b \in D$, $\delta(a) < \delta(ab)$ if and only if b is not a unit.

(F) Show that $\mathbb{Z}[\sqrt{10}]$ is not a unique factorization domain.