Solutions

MATH 314–Linear Algebra
Quiz 1
Dr. Janet Vassilev

(1) For the following system of equations, use back substitution to solve for (x_1, x_2, x_3):

$$x_1 - 2x_2 + 7x_3 = 8 \Rightarrow x_1 = 8 + 2(5) - 7 \cdot 2 = 4$$
$$x_2 - x_3 = 3 \Rightarrow x_2 = 3 + 2 = 5$$
$$x_3 = 2$$

$$(x_1, x_2, x_3) = (4, 5, 2)$$

(2) Consider $A = \begin{pmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$.

(a) Which of the matrices are in row echelon form? B & C

(b) Which of the matrices are in reduced row echelon form? C

(c) For each matrix not in reduced row echelon form, decide if you can perform one row operation to put it into reduced row echelon form. Indicate the matrix and the row operation which would put it into reduced row echelon form.

For A swap rows 1 & 2 (type I)
For B add −2 times row 2 to row 1. (type III)

(3) If $A = \begin{pmatrix} 1 & -2 & 4 \\ 2 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 & -5 \\ 1 & 4 & -7 \end{pmatrix}$, determine $A + B$ and $5A$.

$$A + B = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 4 & -4 \end{pmatrix} \qquad 5A = \begin{pmatrix} 5 & -10 & 20 \\ 10 & 0 & 15 \end{pmatrix}$$

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Quiz 2
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(1) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Determine A^2, A^3 and A^n for $n \geq 4$.

$A^2 = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, $A^3 = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 0 & 0 & 0 \end{pmatrix}$, $A^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} & 2^{n-1} \\ 0 & 0 & 0 \end{pmatrix}$

(2) Let A be any $n \times n$ matrix. Using properties of transposes, show that $A + A^T$ is symmetric.

Set $B = A + A^T$. $B^T = (A+A^T)^T \underset{\text{"sum of transposes"}}{=} A^T + (A^T)^T \underset{\text{"taking transposes twice"}}{=} A^T + A \overset{\text{"Commutative +"}}{=} A + A^T = B$

So $\underset{A+A^T}{B}$ is symmetric

(3) Find the inverses of the following matrices $A = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

$A^{-1} = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad B^{-1} = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(4) Given that $A^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, find the solution to the system $A\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

$\bar{x} = A^{-1}\begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Solutions

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Quiz 3
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(1) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 3 \\ 1 & 4 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.
Find $|A|$, $|B|$, $|C|$ and $|AC|$.

$|A| = 1 \cdot 4 \cdot 5 = 20$

$|B| = 0$ since $C_1 = C_3$

$|C| = -1$

$|AC| = |A||C| = 20(-1) = -20$

(2) Let A be any $n \times n$ matrix. If $\det(A) = \frac{2}{5}$, what is $\det(A^{-1})$?

$\det(A^{-1}) = 5/2$ since

$1 = \det(I) = \det(AA^{-1})$

$= \det A \det A^{-1} = 2/5 \det A^{-1}$

(3) For the matrix $A = \begin{pmatrix} 2 & 2 & 4 \\ 3 & 3 & 5 \\ 0 & 1 & 0 \end{pmatrix}$ find adj(A), $\det(A)$ and A^{-1}.

$\operatorname{adj} A = \begin{pmatrix} -5 & 4 & -2 \\ 0 & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix}$

$\det A = -1(2 \cdot 5 - 4 \cdot 3) = 2$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -5 & 4 & -2 \\ 0 & 0 & 2 \\ 3 & -2 & 0 \end{pmatrix}$