

**Math 421 – Spring 2014**  
**Homework 9**

1. Describe all extensions of the identity map of  $\mathbb{Q}$  to an isomorphism mapping  $\mathbb{Q}(\sqrt[3]{2})$  onto a subfield of  $\overline{\mathbb{Q}}$ .
2. Describe all extensions of the automorphism  $\psi_{\sqrt{3}, -\sqrt{3}}$  of  $\mathbb{Q}(\sqrt{3})$  to an isomorphism mapping  $\mathbb{Q}(i, \sqrt{3}, \sqrt[3]{2})$ .
3. Find the splitting field of  $x^4 - 5x^2 + 6$ .
4. Find the splitting field of  $x^4 + 1$ .
5. Show that if  $[E : F] = 2$ , then  $E$  is a splitting field over  $F$ .
6. Show that for any prime  $p$  the splitting field over  $\mathbb{Q}$  of  $x^p - 1$  is of degree  $p - 1$ .
7. Let  $F$  be any field and  $n$  be any positive integer not divisible by  $p$ , the characteristic of  $F$ . Show that the splitting field of  $x^n - a$  for any  $a \in F$  is  $F(\alpha, \omega_n)$  where  $\alpha$  is any root of  $x^n - a$  and  $\omega_n$  is a primitive  $n$ th root of unity.