Math 421 – Spring 2014
Homework 9

1. If $\alpha, \beta \in \overline{F}$ are separable over F, show $\alpha \pm \beta$, $\alpha\beta$ and α/β (assuming $\beta \neq 0$) are all separable over F.

2. Prove that if E is an algebraic extension of a perfect field F, then E is perfect.

3. Let E be an algebraic extension of a field F. Show that the set of all elements in E that are separable over F forms a subfield of E.

4. Let F be a field and let $f(x) = a_0 + a_1x + \cdots + a_nx^n \in F[x]$. The derivative $f'(x)$ is the polynomial $f'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1}$. Prove that $D : F[x] \to F[x]$ defined by $D(f(x)) = f'(x)$ is a homomorphism and determine the kernel of D both in the characteristic 0 setting and the characteristic p setting.

5. Using the setup from the previous problem. Show the following "derivative" rules: $D(af(x)) = aD(f(x))$ for any $a \in F$, $D(f(x)g(x)) = f(x)D(g(x)) + g(x)D(f(x))$ and $D(f(x)^m) = mD(f(x))f(x)^{m-1}$.

6. Let $f(x) \in F[x]$ and $\alpha \in \overline{F}$ be a zero of $f(x)$. Show that the multiplicity of α is greater than 1 if and only if α is also a zero of $f'(x)$.

7. Show that an irreducible polynomial over a field of characteristic p is not separable if and only if each exponent is divisible by p.

8. Find the separable closure of $\mathbb{Z}_3(y^{12}, z^{18})$ in $\mathbb{Z}_3(y, z)$.