Math 421 – Spring 2014
Homework 2

1. Determine if the following polynomials are reducible or irreducible over $\mathbb{Q}[x]$.
   (a) $x^4 + 4$.
   (b) $x^3 + 3x^2 - 8$.
   (c) $x^5 + 4x^3 + 6x^2 - 18$.
2. Determine if the following polynomials are reducible or irreducible over $\mathbb{Z}/7\mathbb{Z}[x]$.
   (a) $x^3 + 2x^2 + 2x + 1$.
   (b) $x^3 + 2x + 3$.
   (c) $x^2 + 2x + 5$
3. Show that for p a prime, the polynomial $x^p + a \in \mathbb{Z}/p\mathbb{Z}[x]$ is reducible for all $a \in \mathbb{Z}/p\mathbb{Z}$.
4. If $a \neq 0$ is a zero of the polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, then $\frac{1}{a}$ is a zero of the polynomial $a_0 x^n + a_1 x^{n-1} + \cdots + a_n$.
5. Prove the rational root theorem: If $\frac{b}{c}$ is a root of $a_n x^n + a_{n-1}x^{n-1} + \cdots a_0$, then c divides a_n and b divides a_0.
6. Prove that if p is a prime element in an integral domain, then p is irreducible.
7. Prove that if p is irreducible in a UFD, then p is prime.
8. Let p be a prime and R be the subring of the rational numbers of the form $\frac{m}{n}$ where m and n are relatively prime and p does not divide n. Show that R is a principal ideal domain.