Nonlinearity Management in Dispersion Managed System *

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We propose using a nonlinear phase shift interferometric converter (NPSIC) (a new device) for lumped compensation of the nonlinearity in optical fibers. The NPSIC is a nonlinear analog of the Mach-Zehnder interferometer and provides a way to control the sign of the nonlinear phase shift. We investigate a potential use of NPSIC for compensation of the nonlinearity to make a dispersion-managed system closer to an ideal linear system. More importantly NPSIC can be used to essentially improve single channel capacity in the nonlinear regime.

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The recent invention[1] and testing[2–5] of the dispersion management technique demonstrated the effectiveness of this approach for high speed communications. Optical pulse dynamics in fiber links with dispersion management are governed by the nonlinear Schrödinger equation (NLS) with periodic coefficients:

$$iu_z + d(z)u_{tt} + \sigma(z)|u|^2u = 0, \tag{1}$$

where z is the propagation distance, u is an optical pulse amplitude, $d(z) \equiv -\frac{1}{2}\beta_2(z)$, $\beta_2(z)$ is a first order group-velocity dispersion, $\sigma(z) = (2\pi n_2)/\big(\lambda_0 A_{eff}(z)\big)$ is the nonlinear coefficient, n_2 is the nonlinear refractive index, $\lambda_0 = 1.55\,\mu m$ is the carrier wavelength, A_{eff} is the effective fiber area that in general case depends on z.

On short scales, the dispersion managed (DM) system is practically linear. Linear transmission in an optical fiber is limited by a nonlinear distance $z < z_{nl} \equiv (\sigma|u_0|^2)^{-1}$, which is determined by the Kerr nonlinearity σ and the characteristic pulse power $|u_0|^2$. The characteristic power cannot be choosen too small in order to maintain appropriate value of the signal-to-noise ratio. It is natural to attempt to extend the scale of the applicability of the linear regime. This can be achieved through use of a new optical fiber with lower Kerr nonlinearity[6,7]. Another obvious approach is "nonlinearity management", which was considered in[8]. However, the semiconductor material waveguides with negative Kerr nonlinearity proposed as an element for compensation of nonlinear phase shift are currently not practical. In this letter we consider lumped compensation of the nonlinearity, the analog of lumped compensation of the chromatic dispersion by means of chirped fiber gratings. We suggest the use of a nonlinear phase shift interferometric converter (NPSIC) as presented in Fig.1. The NPSIC consists of a silica-based fiber 1, a highly nonlinear fiber 2 (which could be chalcogenide glass based, having a Kerr coefficient about 400 times or even much more than that for silica, see e.g. references[9,10]), a linear amplifier A with amplitude amplification coefficient G, and two directional couplers 1,2. It is assumed that an optical terminator is installed at the end of the fiber 2 after coupler 2 to prevent beam reflection from the end of fiber 2. The incident light with amplitude Gu_0 is split by the directional coupler 1 into two beams with amplitudes a_1Gu_0 and $a_2Gu_0e^{i\pi/2}$ ($a_1 > 0,\ a_2 > 0$), corresponding to the fibers 1 and 2, respectively. We assume that total power is conserved: $a_1^2 + a_2^2 = 1$ but a subsequent consideration can be easily generalized to include the directional coupler's and fiber's losses. The extra phase $\pi/2$ in the second fiber is due to light spliting in the directional coupler (see e.g. reference[11]). The optical lengths $l_1,\ l_2$ in fibers 1 and 2 between couplers 1 and 2 are chosen in such a way that they provide a zero phase difference at the input of coupler 2: $n_{eff,1}l_1 - n_{eff,2}l_2 = 0$, where $n_{eff,1}, n_{eff,2}$ are effective linear refraction indexes in fibers 1,2. We assume that l_1, l_2 are small enough so that we can neglect the influence of dispersion in both fibers and the nonlinear phase shift in fiber 1. Thus the amplitudes of optical beams at the input of the coupler 2 are given by a_1Gu_0 and $a_2Gu_0e^{i\pi/2+i\phi_{nl}}$, where the nonlinear phase shift $\phi_{nl} = \sigma_{hnl}a_2^2G^2|u_0|^2l_2$ and σ_{hnl} correspond to the value of σ in the highly nonlinear fiber 2. Assuming that the propagation constants are the same for symmetric and antisymmetric modes of the coupler, the coupled-wave equation describing mode evolution in directional couplers can be written as[11]

$$(u_1)_z = i\kappa u_2, \quad (u_2)_z = i\kappa u_1, \tag{2}$$

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where

$$u_1(z_0) = a_1 G u_0,\ u_2(z_0) = a_2 G u_0 e^{i\pi/2 + i\phi_{nl}},$$
$$u_{1\,out} = u_1(z_{out}), \qquad (3)$$

z_0, z_{out} are the coordinates of coupler 2 input and output respectively; $u_{1\,out}$ is the optical beam amplitude in fiber 1 at the exit of the coupler 2 and κ is a coupling coefficient which is assumed to be real in the lossless model. The solution of (2), (3) shows that NPSIC converts the u_0 into the output signal $u_{1\,out}$ as follows

$$u_{1\,out} = G u_0 \big(a_1 \cos\phi_c - a_2 e^{i\phi_{nl}} \sin\phi_c\big), \qquad (4)$$

where $\phi_c = \kappa(z_{out} - z_0)$. We assume that $\phi_{nl} \equiv \sigma_{hnl} a_2^2 G^2 |u_0|^2 l_2 \ll 1$ and expand $e^{i\phi_{nl}}$ in the Eq. (4). Neglecting $O(\phi_{nl}^2)$ terms results in

$$u_{1\,out} = u_{lin}\big(1 - \frac{i\sigma_{hnl} a_2^3 G^2 |u_0|^2 l_2 \sin\phi_c}{a_1\cos\phi_c - a_2 \sin\phi_c}\big), \qquad (5)$$

where $u_{lin} \equiv G u_0 (a_1 \cos\phi_c - a_2 \sin\phi_c)$ is the output amplitude $u_{1\,out}$ of the linear system. Thus by changing the NPSIC parameters a_1, ϕ_c, l, G, one can control the sign and magnitude of the nonlinear phase shift. NPSIC can be considered as a nonlinear version of the Mach-Zehnder interferometer. As a typical example we take $G = \sqrt{50}$, $a_1 = 1/3$, $\phi_c = 0.2$ which results in $u_{1\,out} \simeq u_0\big(1.0 - i60.\,\sigma_{hnl}|u_0|^2 l_2\big)$. Thus to completely compensate a nonlinear phase shift $\sigma|u_0|^2 L_1$ in $L_1 = 40\,km$ line of a silica fiber we need to use NPSIC with $l_2 = 1.6\,m$ of a highly nonlinear fiber provided that $\sigma_{hnl}/\sigma = 400$ (for this value of σ_{hnl} the nonlinear absorption is negligible in currently available chalcogenide glasses[10]).

This estimate is true if the power $|u|^2$ is constant throughout the propagation. In practice, power is not constant, due to fiber chromatic dispersion and losses. Therefore, nonlinearity compensators must be distributed along the fiber with a separation distance less than the dispersion length $z_{disp} \equiv \tau^2/d$ and loss length z_{loss}, where τ is a typical pulse amplitude. It is shown below that the effective length of the nonlinearity is increased as $z_{eff,\,nl} \sim N^2 z_{nl}$, where N is the number of lumped NPSIC on the dispersion map period. However, we demonstrate that for short pulses corresponding to strong dispersion management this approach is efficient only at relatively large value of N. Nevertheless, this method displays good performance in the nonlinear regime, as, for example, inserting only two compensating elements on the period of the dispersion map could increase the bit rate per channel by a factor of two.

Consider DM system with step-wise periodical dispersion variation: $d(z) = d_0 + \tilde{d}(z)$, $\tilde{d}(z) = d_1$ for $0 < z + mL < L_1$ (standard monomode fiber) and $\tilde{d}(z) = d_2$ for $L_1 < z + mL < L_1 + L_2$ (dispersion compensating fiber). Here d_0 is the path-averaged dispersion, d_1, d_2 are the amplitudes of dispersion variation subject to a condition $d_1 L_1 + d_2 L_2 = 0$, $L \equiv L_1 + L_2$ is a dispersion compensation period and m is an arbitrary integer number. $\sigma(z) = \sigma_1$ for $0 < z + nL < L_1$ and $\sigma(z) = \sigma_2$ for $L_1 < z + nL < L_1 + L_2$ corresponding to standard monomode and dispersion compensating fiber. We suppose that NPSIC units are located inside and at the ends of a dispersion compensated fiber at points $z_n = mL + L_1 + nL_2/N$, $n = 0, \ldots, N$, $z_0 = mL + L_1 < z_1 < \ldots < z_N = (m+1)L$. At these points the value of $u(t,z)$ experiences jump according to the Eq. (5):

$$u(t,z)\big|_{z=z_{m,n}+0} = \big(u - i z_{eff,n} |u|^2 u\big)\big|_{z=z_{m,n}-0}, \qquad (6)$$

where $z_{m,n} \equiv mL + L_1 + nL_2/N$; $z_{m,n} - 0$ and $z_{m,n} + 0$ mean the coordinate value just before and after the jump, respectively; the parameters a_1, ϕ_c, l, G are chosen in such a way in order to to provide $u_{lin} = u(t,z)|_{z=z_{m,n}-0}$ and $\sigma_{hnl} a_2^3 G^3 l_2 \sin\phi_c \equiv z_{eff,n} \equiv (\sigma_1 L_1 + \sigma_2 L_2)/N$ for n=1,..., N-1 and $z_{eff,0} \equiv z_{eff,N} \equiv (\sigma_1 L_1 + \sigma_2 L_2)/(2N)$. Here the term $-(\sigma_1 L_1 + \sigma_2 L_2)$ provides the compensation of the nonlinear phase shift both in standard monomode and dispersion compensating fibers.

Assuming that the nonlinearity is small $z_{nl} \gg z_{disp}$, $z_{nl} \gg L$, where L is a dispersion map period, one can express u in Fourier domain as a product of an exact solution of linear part of the Eq. (1), corresponding to mean-free dispersion $\tilde{d}(z)$, on a slow function $\hat{\psi}(\omega, z)$: $\hat{u}(\omega,z) \equiv \hat{\psi}(\omega,z) e^{-i\omega^2 \int_{z_0}^{z} \tilde{d}(z')dz'}$ (see reference[12]), where $\hat{u}(\omega,z) = \int_{-\infty}^{\infty} u(t,z) e^{\imath\omega t} dt$. $\hat{\psi}$ is a slow function of z on a scale L which allows to integrate the Eq. (1) over the period L neglecting the slow dependence of $\hat{\psi}$ on z and we get by taking into account the jumps in (6):

$$\begin{aligned} &i\hat{\psi}(\omega,(m+1)L) - i\hat{\psi}(\omega,mL) - L\omega^2 d_0 \hat{\psi}(\omega,mL) \\ &+ \frac{1}{(2\pi)^2}\int \hat{\psi}(\omega_1,mL)\hat{\psi}(\omega_2,mL)\hat{\psi}^*(\omega_3,mL) \qquad (7) \\ &\times K_{tot}(\triangle)\delta(\omega_1+\omega_2-\omega-\omega_3)d\omega_1 d\omega_2 d\omega_3 = 0, \end{aligned}$$

where $s \equiv d_1 L_1$, $\triangle \equiv \omega_1^2 + \omega_2^2 - \omega^2 - \omega_3^2$ and $K_{tot}(\triangle) \equiv K_1(\triangle) + K_2(\triangle)$. A kernel $K_1(\triangle)$ is equal to the usual kernel of the path-averaged equation[12]:

$$K_1(\triangle) = \sigma_L \sin x/x,\ x \equiv s\triangle/2,\ \sigma_L \equiv \sigma_1 L_1 + \sigma_2 L_2, \qquad (8)$$

while $K_2(\triangle)$ accounts for the jumps in (6):

$$K_2(\triangle) = -\sigma_L[\sin x \cot(x/N)]/N. \qquad (9)$$

Suppose that Full Width at Half Maximum (FWHM) τ of $\psi(0,t)$ is subject to the condition $s/(2N\tau^2) \ll 1$ then $K_{tot}(\triangle)$ can be rewritten as:

$$K_{tot}(\triangle) = \sigma_L(\sin x/x)\Big[x^2/(3N^2) + O\Big(x^2/N^2\Big)\Big]. \quad (10)$$

Thus comparing (10) with the kernel (8) of the usual path-averaged equation we get an extra small factor $\left(\frac{s\triangle}{2N}\right)^2/3 \sim L^2/(Nz_{disp})^2$ provided that N is big enough to ensure the condition $\frac{L}{Nz_{disp}} \ll 1$. For short pulses N should be a big number to make the system close to the linear one. Figure 2 shows the dependence of the FWHM τ_{out} obtained after propagation of the initial zero-chirp Gaussian pulse with $\tau_{ini} = 10\,ps$ over a typical transoceanic distance $10^4\,km$ on the number of NPSIC units $N+1$. This dependence is obtained by numerical intergation of the NLS equation (1). The pulse is lunched to the DM fiber at $z = L_1/2$ with the peak power $|u|^2 = 2\,mW$. DM system parameters are $d_0 = 0,\ d_1 = 10.0\,ps^2/km,\ d_2 = -101.6\,ps^2/km, L_1 = 40\,km,\ L_2 = -d_1L_1/d_2,\ \sigma_1 = 0.0013\,(km\,mW)^{-1},\ \sigma_2 = 0.00405\,(km\,mW)^{-1}$. It is seen that the system can be approximately considered linear for $N \gtrsim 30$ in which case the nonlinear correction to τ_{out} is less 5%.

A strong variation of τ_{out} in Fig. 2 for small N is due to the attraction of the initial Gaussian pulse to soliton solution $\hat\psi(\omega, mL) \equiv \hat\psi_0(\omega)e^{imL\lambda}$ of path-averaged Eq. (7). Here λ is a soliton propagation constant. We refer to this soliton solution as the modified DM soliton by analogy with the DM soliton, that corresponds to the usual path-average equation (without nonlinearity management). For $K_2(\triangle) = 0$ we recover the usual DM soliton[13] of the path-averaged equation[12] where the DM soliton width τ_{DM} depends on s only for $d_0 \to 0$ (see e.g. reference[14]). For the above mentioned system parameters $\tau_{DM} \simeq 21ps$. Any shorter pulses experience strong distortion because of the nonlinearity for $K_2(\triangle) \equiv 0$. Curve 2 in Fig. 3 shows the pulse power distribution after 10^4km of propagation of the initial Gaussian pulse with $\tau_{ini} = 10\,ps$ (curve 1). Using NPSIC units we can control $K_2(\triangle)$ and thus change τ_{MDM} of MDM soliton. Curve 3 shows the pulse power distribution after $10^4\,km$ for a system with two NPSIC for each DM period L located at the ends of the dispersion compensating fiber with $z_{eff,0} = z_{eff,1} = f(\sigma_1L_1 + \sigma_2L_2)/2$, where f=3/2. One can see that curve 3 is close both to the initial Gaussian pulse (curve 1) and to the MDM soliton (curve 4). The MDM soliton was obtained by numerical iteration of the equation (7) with $\hat\psi(\omega, mL) = \hat\psi_0(\omega)e^{imL\lambda},\ \lambda = 0.00028\,km^{-1}$. The numerical iteration scheme is similar to the one used in reference[15] for usual path-averaged equation. By changing the factor f one can control τ_{MDM} of the MDM soliton. Note that for $f > 1$, which corresponds to a negative average nonlinearity of the total system, the numerical iteration scheme finally diverges indicating that the MDM soliton is a rather long-lived quasi-stable structure which however serves as an attractor of the pulse dynamics on a scale $\sim 10^4\,km$.

In summary, the use of NPSIC elements would allow one to construct a nearly linear fiber transmission system. However this system requires many NPSIC elements. On the other hand the use of NPSIC elements offers the better advantages to fiber links operating in nonlinear regime. In particular, only a few elements per dipersion map period could dramatically reduce pulse width and increase bit-rate.

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Figure captions:

Fig.1. Schematic of the nonlinear phase shift interferometric converter.
Fig.2. FWHM τ_{out} after pulse propagation over 10^4 km versus a number $N+1$ of NPSIC units.
Fig.3. Power distributions: an initial Gaussian pulse (curve 1); result of pulse propagation over 10^4 km in a DM system with no NPSIC (curve 2); result of pulse propagation in a DM system with two NPSIC units for each period L (curve 3); the MDM soliton (curve 4).

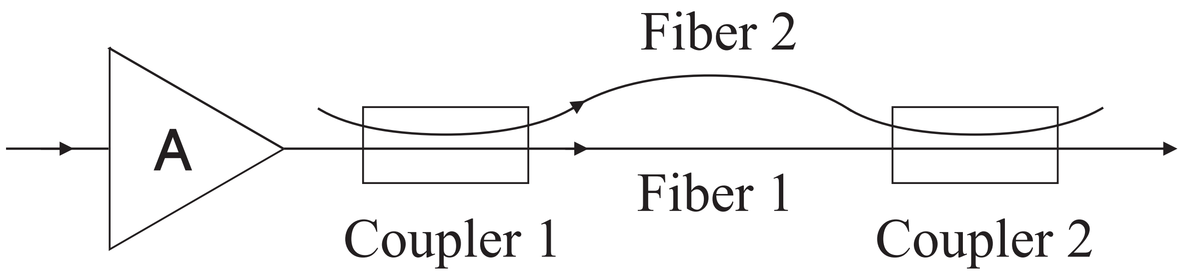

Figure 1

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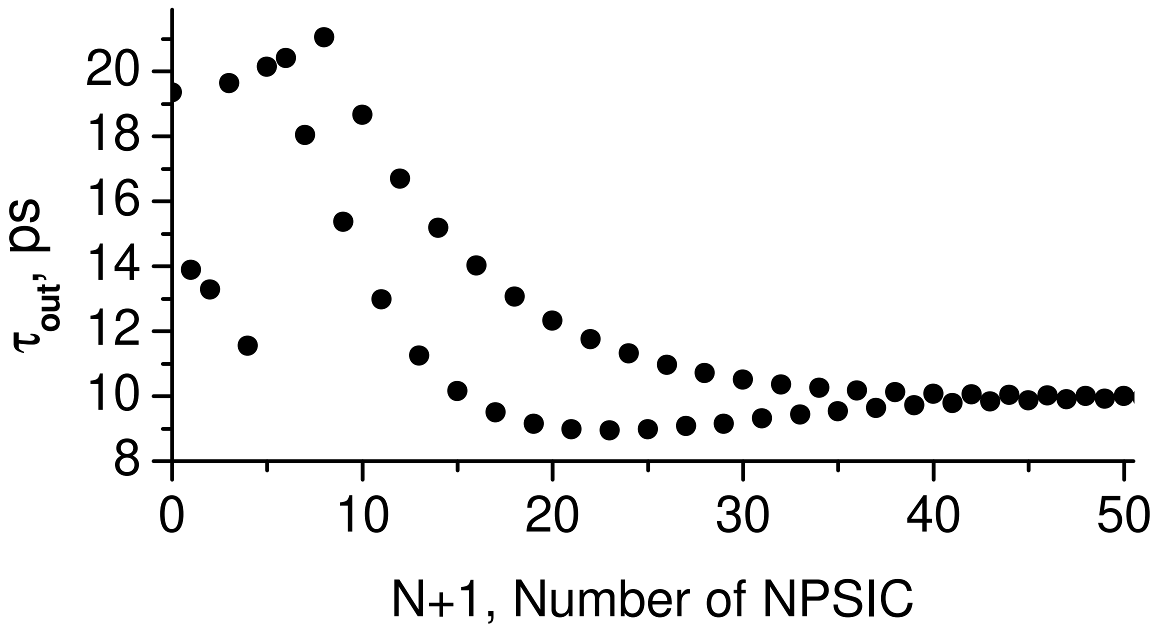

Figure 2
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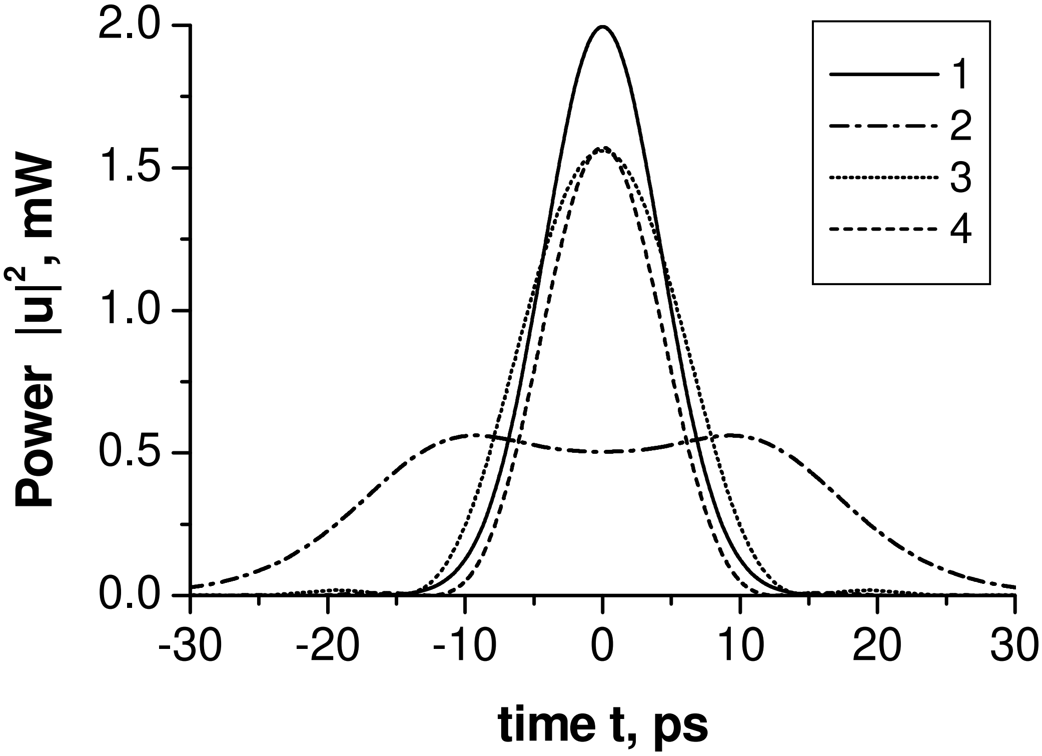

Figure 3.
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