Symposium: Nonlinear Waves and Singularities in Optics, Hydrodynamics and Plasmas

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Progress in the development of new tools for modern applied mathematics resulted in a better scientific understanding of nonlinear waves in various fields of nonlinear optics, hydrodynamics, and plasmas. The universality of the equations describing wave processes is one of the most important components of this success. Nonlinear Schrödinger equation (NLS) and Korteweg de Vries equation are among best illustrations of successful application of applied mathematics as a tool to analyze various nonlinear phenomena ranging from optical communications and Bose condensation to ocean waves. Solutions of nonlinear equations usually result in the formation of singularities, coherent structures or solitary waves. Examples of the corresponding phenomena can be observed in filamentation of laser beams in nonlinear media, wave breaking in hydrodynamics, collapse, and Langmuir waves in plasmas.

NLS in dimension one (1D) is a basis for the description of modern optical fiber communication systems. A key technological tool for development of ultrafast high-bitrate optical communication lines is a dispersion management. A dispersion-managed optical fiber is described by NLS with periodic variation of dispersion along an optical line which dramatically reduces pulse broadening. Solitary wave solutions in such systems are important information carriers in optical lines.

NLS in 2D describes a stationary self-focusing of light in optical media with Kerr nonlinearity, Bose-Einstein condensate in plane geometry as well as a self-focusing in plasma. Due to self-focusing the amplitude of an NLS solution tends to infinity after finite distance of propagation, representing a blow up phenomenon. The blow up is accompanied by a dramatic contraction of the spatial size of solution, which is called collapse. Near the collapse point NLS loses its applicability and there is usually a qualitative change in the underlying nonlinear phenomena, the new mechanisms become important such as dissipation, inelastic two- and three-body collisions, which can cause a loss of atoms from the Bose-Einstein condensate, breakdown of slow envelope approximation in nonlinear optical media and plasma density depletion. If optical power is very large in optics or if a number of particles in the Bose-Einstein condensate significantly exceeds the critical number, then collapse turbulence occurs with random positions of collapses in space and time. A study of collapse regularization is important in describing collapse turbulence and to be able to go beyond an individual collapse event.

3D NLS is a model for non-stationary self-focusing of light in optical media with Kerr nonlinearity, Bose-Einstein condensate in 3D geometry and laser-plasma interactions. Self-focusing results in collapse, which is of a different type as compare with 2D, and it is called weak collapse. NLS in 3D is a model for ultrashort pulse (optical bullet) that is a key for current developments in microfabrication of photonics devices by means of intense femtosecond laser pulses. Regularization of collapse for such pulses requires an extension of the model beyond NLS and a study of the full set of MaxwellŠs equations. Maxwell's equations are also necessary to study ultrashort light propagation through thin films and guided-wave metamaterials.

New challenges for nonlinear optics and applied mathematics include description of nonlinear optical phenomena in negative-refractive-index metamaterials, which are artificial nano-composites materials that show great promise for opening the new avenues in manipulating light. Light propagating through them could exhibit extraordinary properties and brings a wealth of novel phenomena to nonlinear optics. Electrodynamical properties of metamaterials result from many factors: plasmonic oscillations in metallic nanostructures, multi-scale interaction of near field electromagnetic resonances with optical far field dynamics, surface roughness of nanostructures, high values of local fields that cause nonlinear response and etc. It has recently been shown that mathematical modeling of light interaction with thin film metamaterials requires the use of classical hydrodynamics models. Therefore, metamaterials represent a new research

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subject that combines nonlinear optics, hydrodynamics and plasma physics.

Accurate computer modeling of the optical properties of metamaterials is another challenging problem emerging from the need to perform 3D multiscale analysis and from the high-contrast optical properties of metallic nanostructures and host dielectric materials. Straightforward application of conventional algorithms requires significant computational resources and is also severely limited by slow convergence due to discontinuities at the boundary metal-dielectric. Interaction of experts in computer modeling in hydrodynamics, plasmas and nonlinear wave theory with researchers in the field of material science represents an excellent opportunity to take an advantage of the extensive expertise in these fields for computer modeling of optical metamaterials. The symposium brings together the leading scientists in these fields combining theoretical and experimental studies.

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