

Symmetry in Newtonian Mechanics

1. Suppose relative to a fixed choice of coordinates (x, y, z) the position of a particle P at time t is given by

$$f(t) = (f_1(t), f_2(t), f_3(t))$$

- a. What is the physical meaning of the derivatives $f'_1(t), f'_2(t), f'_3(t)$?
- b. Suppose $f'_2(t) = f'_3(t) = 0$ for all t . What can you say about the particle P ?
- c. What is the physical meaning of $f''_1(t), f''_2(t), f''_3(t)$?
- d. What can you say about the particle P if all three second order derivatives in part **c** are zero?
- e. Find an equation $f(t)$ for a particle which starts in the x - y plane and then spirals upward counterclockwise around the z -axis: for simplicity, suppose the distance from the z -axis and the velocity remain constant. What are the partial derivatives of f ? Explain the physical meaning of these derivatives. How do they change if the particle moves around clockwise instead of counterclockwise?

2. Suppose you choose a new set of coordinate axes with origin located, in the old coordinate system, at (a, b, c) .
- a. What is the equation $g(t)$ for the position of the particle P at time t *relative* to this new coordinate system?
 - b. How do the answers to **b** through **d** in problem **1** change if the computations are made relative to the new coordinate system?
 - c. What does this tell you about the symmetry of Euclidean space with regards to the notions of velocity and acceleration?

3. Suppose, instead of translating the origin, the origin is left fixed but the x, y, z coordinate system is *rotated* about the origin. For simplicity, assume that $z' = z$, so that the z -axis is left fixed and that the (x, y) axes are rotated through θ degrees, counterclockwise, to obtain the new coordinates x', y' .
- Suppose (a, b, c) is a point in \mathbf{R}^3 given relative to the (x, y, z) coordinate system. What are the coordinates of (a, b, c) relative to the (x', y', z') system?
 - What type of map is the map $f : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ given by $f(x, y, z) = (x', y', z')$?
 - What is the equation for f^{-1} ? In other words, given a point in (x', y', z') -coordinates, what are the (x, y, z) coordinates? How would you express f and f^{-1} as matrices?
 - Suppose instead of x, y one uses polar coordinates (r, θ) . How do the equations for f and f^{-1} change? Are they simpler or more complex? Which would you choose to use if you had a lot of calculations to do?

4. Here we will study Newton's fundamental law

$$F = ma.$$

Suppose P is some particle moving in \mathbf{R}^3 with its position at time t given by $P(t) = (f_1(t), f_2(t), f_3(t))$. Here F denotes the force acting on P , m is its mass, and a is its acceleration.

- a.** Suppose the coordinates (x, y, z) are translated to obtain new coordinates (x', y', z') . Does the equation $F = ma$ remain invariant under this coordinate change?
- b.** Suppose the coordinates (x, y, z) are rotated counterclockwise about the z -axis to obtain new coordinates (x', y', z') . Does the equation $F = ma$ remain invariant under this coordinate change?
- c.** Parts **a** and **b** show that Newton's law is invariant under distance and orientation preserving transformations of Euclidean space. Is this to be expected? What would be the ramifications, for physics, if this translation and rotation invariance were false?