Galilean Invariance

Newton's law holds not only with respect to any coordinate system in $\mathbf{R}^3$ but also with respect to any *constantly* moving reference frame. Galileo wrote, in 1632,

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you some flies, butterflies, and other small flying animals . . . hang up a bottle that empties drop by drop into a wide vessel beneath it... Have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that . . . The droplets will fall into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans . . . the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship.

For simplicity, we fix two closely related reference frames:

i. the normal (x, y, z)–axes at rest,

ii. the normal (x, y, z)–axes moving at constant velocity v_0 in the direction of the positive x–axis.

Suppose a particle P is travelling in the direction of the positive x–axis with position $P(t)$, *relative* to coordinate system **i**, at time t. Suppose the velocity of P, relative to the (x, y, x)–axes at rest, is constant and equal to v.

1. What will be the observed velocity of P for a person with reference frame **ii**? Explain what happens in the three cases $v_0 < v$, $v_0 = v$, $v_0 > v$. Check that Newton's equation $F = ma$ remains invariant with respect to the moving reference frame.

2. Suppose we choose (x', y', z') as on problem **3** on the previous problem set so that (x', y', z') represents the (x, y, z)–axes rotated θ degrees counterclockwise around the z–axis. Show that the equation $F = ma$ still holds along the x'–axis (note: the force acting on P is in the direction of the x–axis so relative to a different coordinate system it will have components in one, two, or all three directions).

3. Would there be any *perceived* difference (by Juanita) between the following two phenomena:

a. Juanita is in a train *at rest* while outside the window a second train moves by travelling in the opposite direction at constant velocity v.

b. Juanita is in a train moving at constant velocity v past a train which is not moving.

Does the answer to this question change if the velocity is *not* constant?

4. To see that there is something serious at issue in **3**, consider the following reference frames for studying the path of a particle P:

a. viewing the earth as fixed (the usual choice),

b. considering only the rotational motion of (an otherwised fixed) earth,

c. considering the orbit of the earth around a (presumed fixed) sun,

d. considering the motion of the sun within a (presumed fixed) galaxy.

Beyond convenience for the calculation/experiment at hand, do any of the points of view listed above distinguish themselves as inherently more natural or valuable than the others? What would the consequences be for physics if the laws of nature *changed* depending upon the framework within which you work?