Complex Analysis
Qualifying Examination

Summer 2003

Instructions: There are eight (8) questions on this examination, and each question is worth 25 points. A maximum score of 200 points is possible.

1. Determine the location and type of singularities of the following functions:
   (i) $\cosh^2(\dfrac{1}{z-\pi})$
   (ii) $z + z^3$
   (iii) $(cosz - sinz)^{-1}$
   (iv) $\tan(\frac{1}{z})$

2. Expand in Laurent series in region indicated:
   $\exp(\dfrac{1}{z-1})$, $|z| > 1$

3. Let $f(z) = z^{10} + \frac{1}{2}z^6 + \frac{1}{100}\exp(z^5)$
   (i) State Rouche's theorem.
   (ii) Show that $f(z)$ has no zeros on $|z| = 1$.
   (iii) How many zeros does $f(z)$ have inside $|z| = 1$? Justify your answer.

4. Assume $f(z)$ is analytic inside and on a contour C and b is a complex number such that $f(z) = b$ for some z on C. What is the significance of

$$\frac{1}{2\pi i}\int_C \frac{f' dz}{f - b}$$

5. State and prove Liouville's theorem.

√6. Given

$$f(z) = \frac{1}{z} - \frac{2}{z^2},$$

find

$$\int_C z^2 \exp(\frac{1}{z}) f(z) dz$$

where C is the unit circle traversed counterclockwise.

7. Evaluate

$$\int_0^\infty \frac{\sqrt{(x)}}{(1+x)^3}\,dx$$

by contour integration.

8. Evaluate

$$\int_{-\infty}^{\infty} \frac{\exp(ikx)}{(1+x^2)}\,dx$$

by contour integration (assume $k > 0$, real).