Geometry/Topology Qualifying Exam Department of Mathematics & Statistics University of New Mexico

Instructions: Do the following problems. Show all your work.

- 1. Let (X, d) be a metric space and S a set. Show that $\mathcal{B}(S:X)$ is a complete metric space iff X is complete. Here, $\mathcal{B}(S:X)$ is the space of bounded functions from S to X with the metric defined by $\rho(f,g) = \sup_{s \in S} d(f(s),g(s))$.
- 2. A topological space X is called locally connected if for each point $x \in X$ and open neighborhood U of x, there is a connected open neighborhood V of x contained in U. Show that a connected component of a locally connected topological space is open.
- 3. a) Show that every smooth submersion map $F: M \to N$ between two smooth manifolds M and N is an open map.
 - b) Show that a surjective submersion F between two smooth manifolds is a quotient map, that is, show that a set in the target space is open iff the pre-image is open.
- 4. a) Give a definition of a smooth manifold to be orientable.
 - b) Show that the tangent bundle of a smooth manifold is an orientable manifold (prove only the orientability claim).
- 5. Let G be the group of counter clock-wise rotations around the z-axis (when viewed from above the xy-plane) in the three dimensional Euclidean space with a fixed Cartesian coordinate system.
 - a) Give a representation of G as a group of 3 by 3 matrices.
 - b) Find the infinitesimal generator of G.
- 6. Let M and N be smooth manifolds and G a Lie group acting smoothly on both M and N, so that the action on M is transitive.
 - a) Show that if $F: M \to N$ is a smooth equivariant map with respect to the action of annihilated by 8 G, i.e., $F(g \cdot p) = g \cdot F(p)$ for all $p \in M$, then F has constant rank.
 - b) Show that the level sets of F are closed embedded submanifolds of M.
- 7. Let $\theta = xdy ydx + dz$ be a one-form on \mathbb{R}^3 .
 - a) Show that the kernel of θ defined as the set of all tangent vectors to \mathbb{R}^{3} is a rank two vector bundle over \mathbb{R}^3 .
 - b) Let H be the vector bundle defined in part a). Is H a trivial vector bundle?
 - c) Is H integrable, i.e., is H the tangent space of a submanifold of \mathbb{R}^3 ?
- 8. Let X be a Hausdorff topological space. Show that points are closed in X.

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- 7. Let let and N besined it mountable and to a Lie group ween smooths on tech at any N so that the entire on life transitive.
- (a) Show that if F wifer N is a suspin equivariant $x \cdot p$ with respect to the action of G, i.e., F as p = -1 by Eq. (b) by all $p \in M$, then F has a costout rank.
 - b. Show that the level sens of I most sequenceded subjuminities of M.
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 - Supplies H to the vector bundle defined of partial. Left a trivial vector curvies F(H) integrable, i.e., is H the tangent space of a submanifold of rescentification.
 - S. Let X us a Third doubt topological shake. Show that points are elected in X.