FALL 1997

Syllabus for the Master's exam Real Analysis

Topics:

- 1. Basic properties of the real numbers: **R** is an ordered field with the l.u.b. property.
- 2. Basic topological concepts defined on a metric space: open and closed sets, compact sets, connected sets, perfect sets, their fundamental properties.
- 3. Sequences and series of real or complex numbers and some convergence tests.
- 4. Real or complex-valued functions, limits, continuity, uniform continuity, differentiability, the Mean Value Theorem, l'Hospital's Rule, Taylor's Theorem with remainder, equicontinuity.
- 5. The integration theories of Riemann and Darboux, the Fundamental Theorems of Calculus.
- 6. Convergence and uniform convergence for sequences and series of functions; continuity, differentiability and integrability of their limits, Stone-Weierstrass theorem, power series, exponential and trigonometric functions defined as power series. the Gamma function.
- 7. The derivative for functions f: $\mathbb{R}^m \to \mathbb{R}^n$ and the basic properties for such derivatives, including the Chain Rule.
- 8. The Inverse Function Theorem and the Implicit Function Theorem.
- 9. The Riemann integral for **R**-valued functions defined on a suitable domain in **R**ⁿ and its basic properties.
- 10. Partitions of unity and the Change-of-Variables Theorem.
- 11. The theorems of Green, Gauss and Stokes.

Most of these topics can be found in the following textbooks. W. Rudin, Principles of Mathematical Analysis, Chapters 1-9. M.H. Protter and C.B. Morrey, A First Course in Real Analysis, Chapters 1-9, 14, 16. J.R. Munkres, Analysis on Manifolds, Chapters 2-7. S.H. Weintraub, Differential Forms, A Complement to Vector Calculus, Chapters I-V.