Computational aspects of the N-body problem

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The study of central configurations in the N-body problem of Celestial Mechanics offers great computational challenges that can be successfully settled using Computer Algebra methods and techniques.

The general aim is to search and classify all possible types of central configurations. This is especially interesting for the planar newtonian 4-body problem with equal masses and for the spatial newtonian 5-body problem with equal masses as well as for various types of corresponding problems with non equal masses.

The fundamental fact that allows us to apply Computer Algebra methods to the problem of central configurations is the description of the equations of central configurations by a system of polynomial equations. A theoretical tool that is used to simplify the problem is the demonstration of the symmetry theorems. The original polynomial equations are processed using the matrix formalism in order to obtain an equivalent but easier to solve system. The algorithmic tools used are resultants, Gröbner bases, Triangular sets and some Invariant theory of Finite Groups. The Computer Algebra programs used are Axiom, Macsyma, Maple, Pari, NTL, Gb, FGb, Magma, MPS, RealSolving and RS.

On the bare computational level the study of central configurations gives rise to zero-dimensional polynomial systems in the case of equal masses. Some of these systems are easy to solve but others are inaccessible. In the case of non equal masses one obtains parametric polynomial systems.

Another interesting computational aspect of the problem is the use of quadratic forms to interpret the determinental identities that arise in the proofs of the symmetry theorems. The study of the corresponding isotropic cones might lead to easier proofs. Since the number of variables of these quadratic forms is more than 10, a Maple package called `QF` has been developed in order to study them.

The study of central configurations gives rise to a wide variety of challenging problems that are suitable for Computer Algebra. The key notion is a better understanding of the symmetries.