Difficulties in solving a nonlinear system of equations using a computer algebra

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Abstract

We study a theoretical mathematical problem which involve the equality of two power series with coefficients depending on four parameters. To determine the values of parameters for which the power series are identical, we have to equate the coefficients of x^k in the given series for $k = 0, 1, ..., n$. So, we have to solve an algebraic system of $n + 1$ equations with four variables, for arbitrary n..

For $n = 5$ the system is of degree three and reduces to the following three equations:

$$u + v = 2 - r - s$$

$$6\left(u^3 + u^2v + uv^2 + v^3\right) + 5(u + v)^2 - 250(u + v)$$
$$= -6\left(r^3 + r^2s + rs^2 + s^3\right) + 15(r + s)^2 + 210(r + s) - 458$$
$$6\left(u^3 + u^2v + uv^2 + v^3\right) + 5(u + v)^2 - 130(u + v)$$
$$= -6\left(r^3 + r^2s + rs^2 + s^3\right) + 15(r + s)^2 + 90(r + s) - 218.$$

Any computer algebra can solve this system obtaining two of the parameters as functions of the other two parameters.

For $n = 6$ we add to the above system an equation of degree five, with a similar structure. His resolution was impossible using computer algebras like Maple 9, Maple 12, or Mathematica 4 (though during more than 24 hours, on computers with 2Gb of memory).

For $n = 8$, a new equation of degree five and an equation of degree seven are added to the system. We expect that either this system has no solution, or it has some numerical solution(s). But it wasn't possible to obtain an answer with any of the mentioned computer algebra systems.

Finally let us present shortly the mathematical problem which leads to the above systems. A mean N is the complementary of the mean M with respect to the mean P if

$$P(M(a, b), N(a, b)) = P(a, b), \forall a, b > 0.$$

We look after Stolarsky means $E_{u,v}$ with the property that their complementary with respect to the logarithmic mean L is again a Stolarsky mean $E_{r,s}$, where

$$E_{r,s}(a,b)=\left(\frac{s}{r}\frac{a^r-b^r}{a^s-b^s}\right)^{\frac{1}{r-s}},rs(r-s)(a-b)\neq 0$$

and

$$L(a,b)=\frac{a-b}{\ln a-\ln b},a\neq b.$$

This problem is of great importance in the study of double sequences of Gauss type.