Polynomial Equations Arising in Global Positioning Systems and in Nash Equilibria

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We discuss systems of multivariate polynomial equations that arise in two important applications. The first is GPS, or global positioning systems. We show that certain equations arising from the 3D affine transformation problem can be completely solved symbolically with the Dixon-EDF method [2]. Other symbolic techniques failed. One of these systems has six equations in six variables and twelve parameters. Another has nine equations in nine variables and eighteen parameters. In both systems, every equation has (total) degree three. I am indebted to Bela Palancz (and his colleagues Piroska Zaletnyik and Joseph Awange) for introducing me to these polynomial systems. See [3].

Secondly, we use Dixon-EDF to solve several sets of equations that arise from the study of Nash equilibria. This is an important topic in economic game theory. We examine the case of four players with two pure strategies each. That produces a set of eight equations with eight variables and thirty-two parameters. Then we look at a system that is not totally mixed (with six equations, six variables, 24 parameters), and lastly at a "cube game" (eight, eight, four). These are all found in the book by Sturmfels [4] and the paper by Datta [1]. Apparently we are the first to provide fully symbolic solutions to these games.

There is a common thread in these two apparently disparate subjects: all the equations are of degree one in each variable. That is, in every equation no variable is squared. In only one of the equations is any parameter squared.

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