Combinatorial integration (Part I, Part II)

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Abstract

Let X be a formal indeterminate. A *combinatorial power of* X is an expression of the form X^n/H where H is a subgroup of the symmetric group S_n. More generally, a *combinatorial power series in* X (CPS, for short) is of the form,

$$\sum_{n,H} c_{n,H} X^n/H, \qquad c_{n,H} \in \mathbb{C}. \tag{1}$$

Many operations have been defined on such series and implemented on computational algebra systems. In particular, CPS form a differential ring, denoted $\mathbb{C}||X||$, equipped with a substitution operation, which contains the ring $\mathbb{C}[[X]]$ of classical power series in X. The main reason to study CPS is that they "encode" classes (species) of combinatorial structures, according to their automorphisms groups, together with the combinatorial operations between them.

In the present talk, we put emphasis on computational techniques for combinatorial integration in $\mathbb{C}||X||$, the inverse of combinatorial differentiation. It turns out that integrals are no longer defined up to a constant. One integral of the class of total orders is the class of oriented cycles; one integral of the class of forests of rooted trees is the class of trees, etc. Integration techniques for families of combinatorial differential equations are also presented and illustrated on explicit examples, including "combinatorial liftings" to $\mathbb{C}||X||$ of the Lambert W function. Various tables computed using Maple and GAP softwares are also included.

Keywords
Combinatorial power series, automorphism groups, species, combinatorial integration