UNWINDING PATHS ON THE RIEMANN SPHERE FOR CONTINUOUS INTEGRALS OF RATIONAL FUNCTIONS

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Abstract. We consider the problem of obtaining integrals of rational functions on domains of maximum extent. We show that for integrals of real rational functions that can be expressed in elementary finite terms, the expressions returned by the Risch algorithm and its variants can have their spurious discontinuities removed using a generalized unwinding number that accounts for both the usual winding of complex functions around 0 in the complex plane as well as windings through the point at infinity on the Riemann sphere. This latter sort of winding occurs where arguments of the arctangent function have poles of odd order. We show that both sorts of windings can be accounted for by introducing, in addition to an *angular unwinding number* that accounts for logarithmic branch cut crossings, a second *radial unwinding number* to account for pole-type singularities, which converts expressions of integrals with spurious discontinuities into correct continuous expressions for the integral. We also discuss the status of an early implementation of the approach in BPAS (basic polynomial algebra subprograms, `bpaslib.org`), an open source, efficient, low-level polynomial algebra software package written in CilkPlus targeting multicore architectures.

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