

Using the Universal Math Environment Math-XPress for teaching and assessment of math courses

Part I, Part II

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In the talk we describe the main features of the Universal Math Environment Math-XPress and its use for classroom teaching, home training and assessment of math students at college and university levels. Math-Xpress includes linked modules of equation editor, 2D and 3D graph plotter, CAS expression evaluator and step-by-step solver, dynamic geometry (2D and 3D) and problem solving tutor.

Using the Problem Generator the courses in Calculus, Linear Algebra, Diff Equations, Statistics, Elementary Algebra, Geometry and others have been developed and used at Talpiot teacher's college and Ariel university regularly since 2007, involving thousands of students each academic year.

Part I

Math-Xpress the Universal Math Environment

The basic module of Math-XPress is *XPress-editor* - a graphical formula editor, enabling natural WYSIWYG editing of math expressions (Fig. 1), which can be either embedded into Word- or other format pages, or used by CAS based *XPress-evaluator*, *XPress-graph plotter* or *XPress-Tutor*.

$$\frac{1}{\varphi} = \frac{2}{\sqrt{5}+1} = \frac{2(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}+1}{2} - 1 = \varphi - 1 \rightarrow \frac{1}{\varphi} = \varphi - 1$$

Fig. 1

Math-Xpress includes also two modules of interactive geometry: *2-D and 3-D XPress-geometry explorer*, which are in turn interrelated to other modules (Fig. 2). *XPress-graph plotter* enables plotting graphs of functions of 2 and 3 variables, families of functions and intersections of graphs (Fig. 3). *XPress-evaluator* performs algebraic operations in final form or step-by-step [1]. The subjects covered by *XPress-evaluator* include: Arithmetic, Elementary Algebra, Trigonometry, Calculus, Probability and Statistics, Linear Algebra, Complex numbers.

It enables to factorize polynomials, to solve equations and systems of equations and to perform most of algebraic operations step-by-step way, or to get numerical solution (Fig. 4).

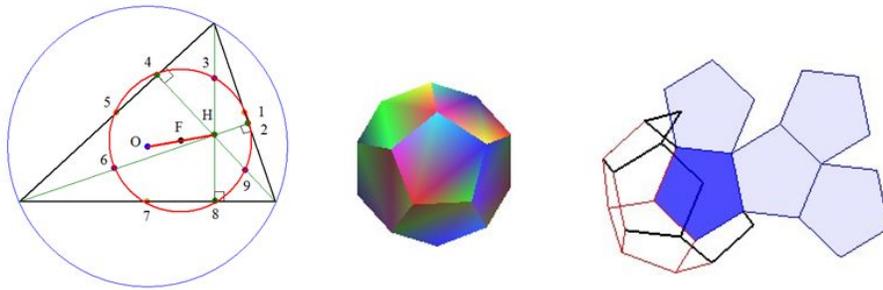


Fig. 2

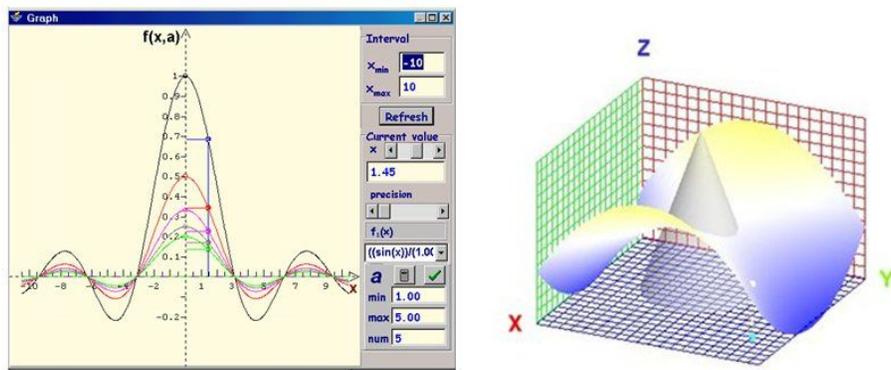


Fig. 3

All the objects created by *XPress-editor*, *Graph Plotter* and *Geometry Explorer* can be imbedded into Word or pdf-pages and called from them directly in interactive Math-Xpress environment. Closing the objects returns to the page from which they have been called.

This technology made it possible to develop fully interactive Math textbooks, first announced at [2] (<https://halomda.com/Maths-5.php>)

Part II

XPress-Tutor consists of content-based problems, presented in three modes: **Learning**, **Training** and **Test**. During a Learning mode, a student is offered a series of problems on a given subject; every problem includes randomly chosen parameters, so that different runs exhibit different initial sets of the parameters.

$$1\frac{3}{5} - 7\frac{2}{9} = 1 - 7 + \frac{3}{5} - \frac{2}{9} = -6 + \frac{3 \cdot 9 - 2 \cdot 5}{45} = -6 + \frac{27 - 10}{45} = -6 + \frac{17}{45} = -5\frac{28}{45}$$

$$\frac{2 \cdot x}{1 - 3 \cdot x} + \frac{4}{5 \cdot x - 6} \qquad \sqrt{x-1} + \sqrt{x+2} = 3$$

$$\frac{2 \cdot x \cdot (5 \cdot x - 6)}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} + \frac{4 \cdot (1 - 3 \cdot x)}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} \qquad 2 \cdot \sqrt{(x-1) \cdot (x+2)} + 2 \cdot x + 1 = 9$$

$$\frac{2 \cdot x \cdot (5 \cdot x - 6) + 4 \cdot (1 - 3 \cdot x)}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} \qquad 2 \cdot \sqrt{(x-1) \cdot (x+2)} = 8 - 2 \cdot x$$

$$\frac{10 \cdot x^2 - 24 \cdot x + 4}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} \qquad 4 \cdot x^2 + 4 \cdot x - 8 = 4 \cdot x^2 - 32 \cdot x + 64$$

$$\frac{2 \cdot (5 \cdot x^2 - 12 \cdot x + 2)}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} \qquad 36 \cdot x = 72$$

$$\frac{10 \cdot x^2 - 24 \cdot x + 4}{(1 - 3 \cdot x) \cdot (5 \cdot x - 6)} \qquad x = 2$$

$$\qquad \qquad \qquad x_1 = 2$$

$$\qquad \qquad \qquad \sqrt{2-1} + \sqrt{2+2} = 3$$

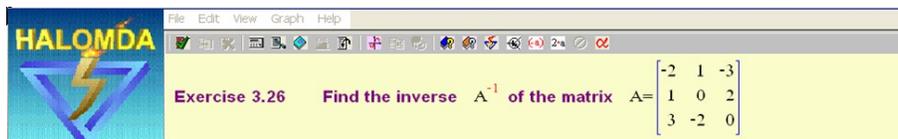
Fig. 4

A student may try solving a problem in his way, by entering an answer or an intermediate step of a solution. The program checks the input expression and responds.

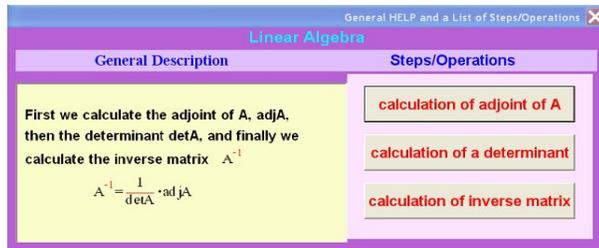
A student can also ask for a Help, that is presented in 3 levels:

- 1) A General Help, where a method of solution common to all the problems of a specific subject is described;
- 2) A List of Steps of a problem solution and the description of every step;
- 3) The Results of every Step of Solution (numerical or algebraic)

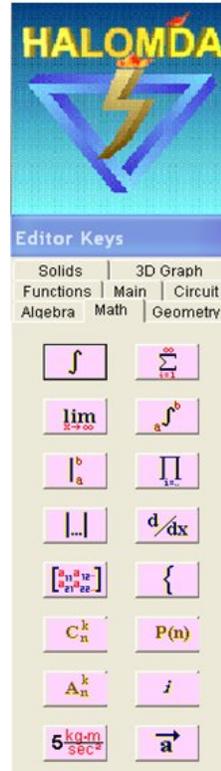
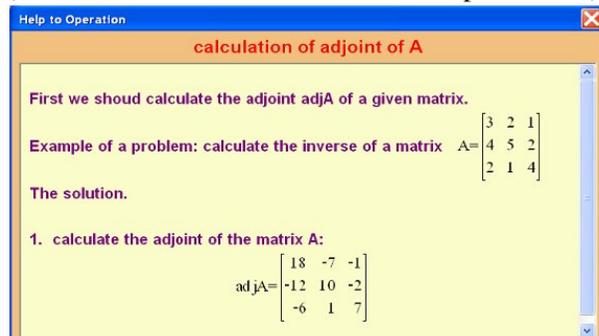
For the demonstration we consider the following example:



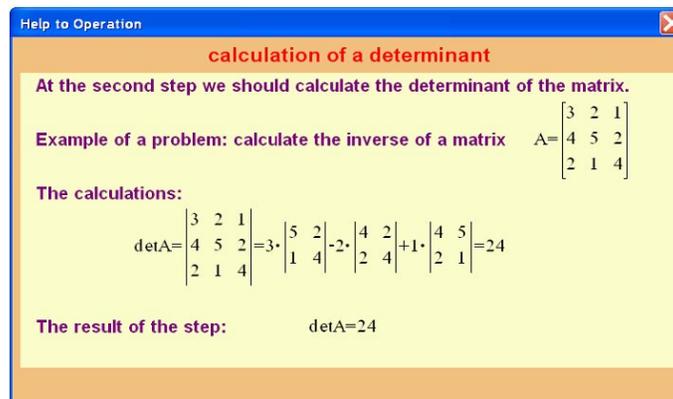
After trying to solve the problems a student can enter his result using the Editing Tools, or, pressing the Help key, he/she can call the Help window, where the General Method and a List of Solution Steps are presented:



The General Method outlines the general ideas and methods that should be applied when solving a problem of a given type. By clicking the keys of solution steps, a student will see the detailed step-by-step solution of a problem similar to that offered to him/her (however, with different initial set of parameters).



After reading the description of a current step, the student is supposed to be able to implement it to the solution of the given problem. If, however, he/she still cannot get the correct result of the step, clicking on **Hint** shows the result. A student may wish to learn how to proceed with the solution, and call for the explanation of the second step:



After finishing all the steps, a student can either move to the next problem, or repeat the current one with a new initial data.

In **Training** mode, instead of viewing the result of every step when clicking **Hint**, a multiple choice of 4 possible results is presented, whereas in the **Test** mode, **no Help** is available, and a student solves the series of the problems as during the regular test.

In both Learning and Training modes **all the features of Math Xpress are available**, so that in a course of problem solving a student can explore the problem using different tools, that can help him/her in better understanding of a solution.

The problems are developed using the *XPress Problem Generator* external module, enabling compiling of new items by unexperienced in programming people [3].

During the last years thousands of problems have been developed covering the courses in Arithmetic, Elementary Algebra and Geometry for primary and intermediate schools, Algebra, Trigonometry and Introduction to Calculus for high schools, and Calculus, Linear Algebra, Differential Equations, Probability and Statistics for universities and colleges.

During the last academic year a course of Quantitative Thinking has been taught for 2 groups of students at Talpiot teacher's college, and all the courses on High Math have been used for teaching and intermediate exams for more than 3000 students at Ariel University.

References

- [1] S.Kornstein, *Xpress Formula Editor and Symbolic Calculator*, Mathematics Teacher **94**, 5, p. 424 (2001).
- [2] P.Slobodsky, *Computerized textbook in Physics and Math - a new approach to science education*, in *The Tenth International Conference on Technology and Education*, Cambridge, USA, pp. 25-26 (1993).
- [3] P.Slobodsky, *Workshop on Integrated Computer Lessons in Physics and Mathematics developed on the basis of program generators "High Class", "Stages" and "Xpress"*, in *The 14 International Conference on Technology and Education*, Oslo, Norwegian, pp. 64-66 (1997).