

Periodic and Nontrivial Periodic Input in Linear ODEs (Part I, Part II)

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Differential equations courses are among the ones where the use of computer algebra systems (CAS) was first experienced. In many cases, it was and it is still for solving application problems where the computation can become very long and tedious. Textbooks as [1] and [2] contain very interesting projects on which students can work. Unfortunately, some authors seem to forget the important role CAS can play in increasing student's understanding of theoretical concepts.

The first part of the talk will be devoted to introduce the subject: we will present the classical problem of finding the steady-state solution of a damped mass-spring problem where the external force is a pure cosine of different frequency. Then, in the second part, the external force will be a nontrivial periodic one. This is well documented but we rarely see different approaches. One approach we will use is the convolution of the input with the impulse response. Another approach will be the use of Fourier series because the linearity of the differential equation allows us to apply the principle of superposition. In both cases, the CAS will work for us, computing the convolution and finding the Fourier expansion.

Finally in the case of underdamping, the shape of the frequency response is best understood and illustrated with the aid of sliders. For this purpose, the TI-Nspire CX CAS software will be used. The notebook [3] contains many examples of how to use it in differential equations.

References

- [1] R. KENT NAGLE; EDWARD B. SAFF; ARTHUR D. SNIDE, *Fundamental of Differential Equations*. Ninth Edition, Pearson, 2018.
- [2] ERWIN KREYSZIG, *Advanced Engineering Mathematics*. Tenth Edition, John Wiley and Sons, 2011.
- [3] GILLES PICARD, *Notes de cours MAT265, Équations différentielles*. Available at <https://cours.etsmtl.ca/seg/gpicard/mat265V2.pdf>, 2017.

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