Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Fractals and tessellations: from K's to cosmology

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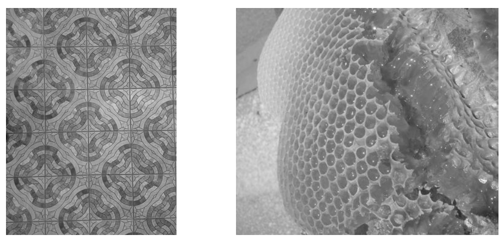
We present two related topics which can accompany Mathematics Education from early childhood to university and even beyond.

1. A *tessellation* is a partition of a space (usually a Euclidean space like the Euclidean plane or the Euclidean 3-dimensional space) by elements of a finite set, called tiles (more precisely, they are non-empty compacts). We begin with considering tilings by translations, i.e. two isometric tiles are deductible from one another by a translation (excluding rotations or symmetries). Generalization is possible to surfaces locally topologically equivalent to a plane. See Figure 1. Another generalization is to accept

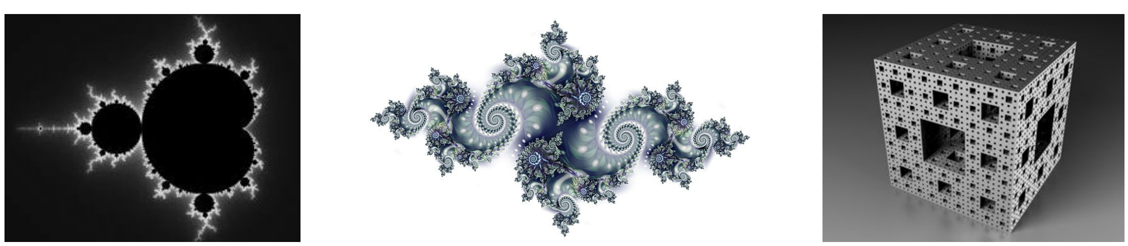


(a) Plane tessellation (b) Honeycomb

Figure 1: Tessellations

symmetries (either central or axial), as the tessellation using a 4^{th} generation Sierpinski triangle in Figure 3 (b).

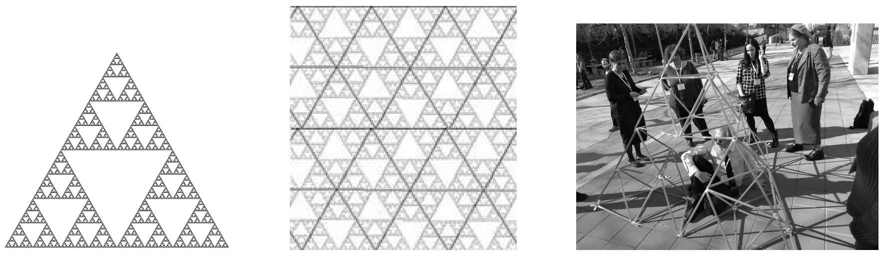
2. A *fractal* is an abstract object used to describe and simulate naturally occurring objects, showing self-similarity at increasingly small scales. The most known example are the *Mandelbrot set* and the *Julia set*. An example of a 3D fractal is the so-called *Menger sponge*. See Figure 2



(a) Mandelbrot set (b) Julia set (c) Menger sponge

Figure 2: 2D and 3D fractals

We show how to use a standard triangular grid to produce tessellation, and how to use a fractal to build a tessellation. Figure 3 shows a Sierpinski triangle, a plane tessellation built with it and a Sierpinski pyramid.



(a) Sierpinski triangle (b) Sierpinski tessellation (c) Sierpinski pyramid

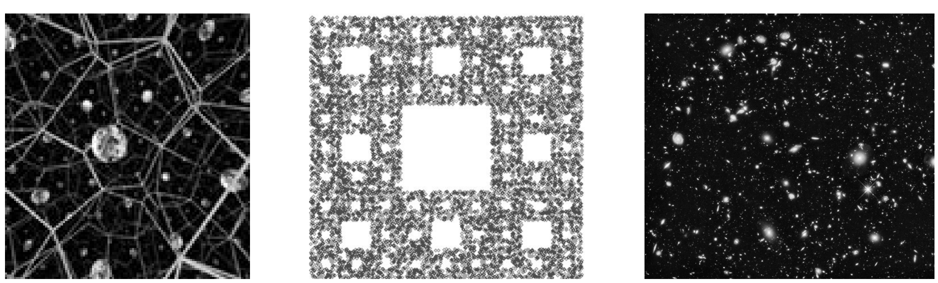
Figure 3: 2D and 3D fractals

Different levels of technology can be used: low-tech such as paper and pencil, then progressive introduction of Dynamic Geometry (in our case GeoGebra: `http://geogebra.org`) and Computer Algebra Systems (we used Maple 2017), together with easy free software to work with images (Irfanview: `http://irfanview.com`).

We demonstrate how to work with technology, using GeoGebra applets, animations* and Maple programming. The examples serving as a basis can come from everyday life and also from more advanced scientific topics, such as the shape of space. In particular, we may quote the following works:

(i) Luminet's theory of *wrapped universe* [1] relies on a kind of 3D tessellation; see Figure 4 (a).

(ii) A work presented at ACA 2017 in Jerusalem in the session on Applied Physics [2], describing the repartitions of galaxies using a Sierpinski gasket; see Figures 4 (b) and (c)†.



(a) Wraparound universe (b) Sierpinski gasket (c) Repartition of galaxies

Figure 4: The shape of space

Part of this exploration has been performed with in-service teachers in a special lab, last February.

*such as `https://www87.homepage.villanova.edu/richard.hurst/Sierpinski.htm#Sierpinski_Carpet`
†Credit: NASA/Hubble

Keywords: Tessellations, Fractals, CAS, DGS

Mathematics Subject Classification 2010: 97U50, 97U70, 97G40, 28A80, 52C15

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[1] J.P. LUMINET, *The wraparound Universe*. AK Peters, 2008.

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