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Teaching Partial Differential Equations with CAS

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Partial Differential Equations (PDE) are one of the topics where Engineering students find more difficulties when facing Math subjects.

A basic course in Partial Differential Equations (PDE) in Engineering, usually deals at least, with the following PDE problems:

1. **Pfaff Differential Equations**, which consists on finding the general solution for:
$$P(x,y,z)\,\mathrm{d}x + Q(x,y,z)\,\mathrm{d}y + R(x,y,z)\,\mathrm{d}z = 0$$
2. **Quasi-linear Partial Differential Equations**, which consists on finding the general solution for: $P(x,y,x)\,p + Q(x,y,z)\,q = R(x,y,z)$ where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.
3. Using **Lagrange-Charpit Method** for finding a *complete integral* for a given general first order partial differential equation: $F(x,y,z,p,q) = 0$.
4. **Heat equation** which consists on solving the second order PDE:
$$\begin{cases} k\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}, \quad k>0 & 0<x<L & t>0 \\ u(0,t)=0 & u(L,t)=0 & t>0 \\ u(x,0)=f(x) & 0<x<L & \end{cases}$$
5. **Wave equation** which consists on solving the second order PDE:
$$\begin{cases} a^2\,\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2} & 0<x<L & t>0 \\ u(0,t)=0 & u(L,t)=0 & t\ge 0 \\ u(x,0)=f(x) & \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = g(x) & 0<x<L \end{cases}$$
6. **Laplace's equation** which consists on solving the second order PDE:
$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 & 0<x<a & 0<y<b \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = 0 & \left.\dfrac{\partial u}{\partial x}\right|_{x=a} = 0 & 0<y<b \\ u(x,0)=0 & u(x,b)=f(x) & 0<x<a \end{cases}$$

In this talk we will describe how we introduce CAS in the teaching of PDE.

The tasks developed combine the power of a CAS with the flexibility of programming with it. Specifically, we use the CAS DERIVE. The use of programming allows us to use DERIVE as a Pedagogical CAS (PECAS) in the sense that we do not only provide the final result of an exercise but also display all the intermediate steps which lead to find the solution of a problem. This way, the library developed in DERIVE serves as a tutorial showing, step by step, the way to face PDE exercises.

In the process of solving PDE exercises, first-order Ordinary Differential Equations (ODE) are needed. The programs developed can be grouped within the following blocks:

- **First-order ODE**: separable equations and equations reducible to them, homogeneous equations and equations reducible to them, exact differential equations and equations reducible to them (integrating factor technique), linear equations, the Bernoulli equation, the Riccati equation, First-order differential equations and nth degree in y', Generic programs to solve first order differential equations.
- **First-order PDE**: Pfaff Differential Equations, Quasi-linear PDE, Lagrange-Charpit Method for First-order PDE.
- **Second-order PDE**: Heat Equation, Wave Equation, Laplace's Equation.

In this talk we will introduce some improvements (redefinition of programs and more types of ODE and PDE) with respect to the talks given in previous ACA [1, 2] related with these topics. We will also remark the conclusions obtained after using these techniques with our Engineering students.

Keywords: ODE, PDE, DERIVE, CAS, PECAS, Engineering

References

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