

Consolidation of abstract knowledge in the process of confronting errors using digital tools: The case of the inflection point

Anatoli Kouropatov¹, Regina Ovodenko²

At the previous ACA conference (see the reference [6]) we reported the results about the development of the integrated teaching unit that was designed towards learning an entire mathematical concept – the inflection point. The unit was built in the digital environment ([3]) and includes geogebra labs, interactive digital questionnaires, and videos, as well as a variety of investigative assignments that are based on them. This environment has been developed with special attention to addressing errors. The development of the environment was informed by research regarding the use of technological tools in math education and research about typical errors in specific mathematical subjects, such as functions ([2]), tangent lines ([1], [7], [9]), inflection ([8]), and so on. We theorized that learning with this unit would allow students to confront errors and to consolidate knowledge about the inflection point. With the purpose of testing this conjecture, we conducted a short feasibility study with a pair of first year students from the Industrial Engineering College. These students are considered advanced students (according to high formal achievements and their lecturer’s personal opinion). It was suggested to the students that they learn the unit after they learned the concept of the inflection point during the course Calculus 1. Their previous encounter with this concept consisted of the part of the process that dealt with investigating functions based on algorithmical usage of well-known theorems related to the concept. The study was organized as a two-hour clinical interview in laboratory conditions. The students’ work was documented and transcribed with the purpose of analyzing their learning process. The analysis of the students’ learning process has been conducted using “Abstraction in Context” (AiC) as developed in [5] as a theoretical framework and as methodological tool ([4]). According to “Abstraction in Context”, learners vertically reorganize previous elements of knowledge to construct new (for the learner) elements and to consolidate previously constructed (by the learner) elements. This construction/consolidation process takes place in students’ minds in a specific context, in our case – learning the inflection point concept using the digital-based teaching unit. In the conference we will present the methodology we used at the study as well as empirical evidence regarding the students’ learning process in general, and regarding the consolidation of abstract knowledge in the process of confronting errors using digital tools, in particular.

References

- [1] M. ARTIGUE, The importance and limits of epistemological work in didactics. In *Proceedings of the 16th Conference of the International Group for the Psychology of Mathematics Education*, W. Geeslin, K. Graham (eds.), Vol.3, 195-216). Durham, NH: University of New Hampshire: PME..

- [2] M. CARLSON, A cross-sectional investigation of the development of the function concept *Research in Collegiate Mathematics III, Issues in Mathematics Education*. **7**(2), 114-162.
- [3] Challenge 5 (2016). Available at <http://lo.cet.ac.il/player/?document=d6beaef0-48a8-4250-b42d-c98cfae422a6>, CET, Israel, 2016.
- [4] T. DREYFUS; R. HERSHKOWITZ; B. SCHWARZ, The nested epistemic actions model for abstraction in context - Theory as methodological tool and methodological tool as theory. In *Approaches to qualitative research in mathematics education: Examples of methodology and methods*, A. Bikner-Ahsbahs; C. Knipping; N. Presmeg (Eds.), 185-217, Dordrecht, Springer: Advances in Mathematics Education Series. Ninth Edition, Pearson, 2018.
- [5] R. HERSHKOWITZ; B. SCHWARZ; T. DREYFUS, Abstraction in context: epistemic actions. *Journal for Research in Mathematics Education*. **32**, 195-222.
- [6] R. OVODENKO; A. KOUROPATOV, The use of digital tools to confront errors. In *Proceedings of 23rd Conference on Applications of Computer Algebra* . ACA 2017.
- [7] D. TALL, Constructing the concept image of a tangent. In *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education*, J. Bergeron, N. Herscovics, C. Kieran (Eds.), Vol. 3, 69-75. Montreal, Canada: PME.
- [8] P. TSAMIR; R. OVODENKO, University students' grasp of inflection points. *Educational Studies in Mathematics*. **83**, 409-427.
- [9] S. VINNER, Conflicts between definitions and intuitions – the case of the tangent. In *Proceedings of the 6th International Conference for the Psychology of Mathematical Educations*, A. Vermandel (Ed.), 24-29. Antwerp, Belgium: PME.

¹Center for Educational Technology and the Levinsky College of Education
Tel-Aviv, Israel
anatoliko@gmail.com

²Center for Educational Technology
Israel
ReginaO@cet.ac.il