

## Analyzing discrete suspended chains using computer algebra

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The mathematical description of the shape of various kinds of suspended chains, cables or funiculars under gravity is well covered in the scientific literature. In this talk we apply computer algebra to analyze, classify and animate suspended discrete chains whose links are “thin straight rods” joining the origin  $O$  to a variable endpoint  $P$  in the closed right half-plane. We use Lagrange multipliers to minimize the potential energy of each chain.

In contrast with the continuous limiting case of the catenary where the suspended chain is given explicitly (up to translation and zoom) as an arc of the hyperbolic cosine, the global shape of such discrete regular suspended chains has no simple explicit expression and falls into 3 classes :

*Concave, Parallel, Convex,*

according to the values of the Lagrange multipliers and the position of endpoint  $P$ . This provides to undergraduate students a stimulating example of the application of Lagrange multipliers and computer algebra methods to analyse a discrete optimization problem.

**Keywords:** Discrete chains, catenary, Lagrange multipliers, computer algebra

**Mathematics Subject Classification 2010:** 05D99, 26B99, 68U05

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