

Visualizations of the nondominated set and the efficient set in multicriteria optimization problems using Mathematica

Włodzimierz Wojas¹, Jan Krupa¹

Multicriteria optimization also known as multicriteria programming is a sub-discipline of operation research. It is taught students in framework of such academic courses as for example: Operation research, Multiobjective optimization, Optimization methods or Mathematical programming. Multicriteria optimization problem has a general form:

$$f(x) = (f_1(x), f_2(x), \dots, f_k(x)) \rightarrow \min/\max$$

subject to $x \in X, X \subset \mathbb{R}^n$

where x is a decision variable vector, X is a feasible set in decision space \mathbb{R}^n , $(f_1(x), f_2(x), \dots, f_k(x))$ is a criterion vector and min or max are understood in accordance with the partial order P in criterion space \mathbb{R}^k . We define: a feasible set Y in criterion space as the image of the set X under $f = (f_1, f_2, \dots, f_k)$, the nondominated set $Y_N = \{y \in Y : \text{there is not } y' \in Y \text{ with } y'Py\}$ and the efficient set $X_E = \{x \in X : f(x) \in Y_N\}$. Many academic books contain visualizations of sets X, Y, Y_N, X_E for some linear functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. It would be more difficult but didactically useful to present these sets also for functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ and $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$. It would rather require computer support using for example CAS programs. In this talk we would like to present a few didactic visualizations of sets X, Y, Y_N, X_E for some functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ and $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ using Mathematica.

Keywords: multicriteria optimization, multicriteria programming, didactics of mathematics, mathematics education, CAS, Mathematica

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¹Department of Applied Mathematics
Warsaw University of Life Sciences (SGGW)
Warsaw Poland
wlozdzimierz_wojas@sggw.pl
jan_krupa@sggw.pl