Math 401/501
Assignment 11, due Thursday, November 21

Problems from Taylor:
§4.3, p.92-93: 2, 5, 7, 15
§4.4, p.98: 2, 4, 5

Also, hand in the following extra problem:

Prove that the function

$$f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable for every $x \in \mathbb{R}$ and write a formula for the derivative function $f'(x)$. Then prove that $f'(x)$ is not continuous at 0.

Notes:

- For #15 in 4.3, just treat the case where $\lim_{x\to b} f'(x) = \infty$. That is, don't worry about the case where $\lim_{x\to a} f'(x) = \infty$, as the argument will be roughly the same as in the other case.
- It may be helpful to recall that Theorem 4.1.10 in text works even when you are considering limits at ∞. In a nutshell, $\lim_{x\to\infty} g(x) = L$ if and only if $\lim_{n\to\infty} g(x_n) = L$ whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence in the domain such that $\lim_{n\to\infty} x_n = \infty$. In 4.3 #5, you will assume that $\lim_{x\to\infty} f'(x) = L$ for some finite L, so to show that $L = 0$ it suffices to find a positive sequence such that $c_n \to \infty$, while $\lim_{n\to\infty} f'(c_n) = 0$.

Not collected:
§4.3, p.92-93: 3, 4, 6, 12
§4.4, p.98: 3, 6-13 (not all of these require you to use L'Hôpital's rule)

Reading: 4.3, 4.4, 5.1