Math 510
Assignment 11, due Wednesday, November 25
(note unusual due date)

As usual, hand in Parts I and II separately:

Part I:

1. Suppose that f is uniformly continuous in $\mathbb{R}$ and K is a continuous function on $\mathbb{R}$ such that $K = 0$ outside $[-1, 1]$ and $\int_{-1}^{1} K(x)\, dx = 1$. Define

$$f_n(x) := n \int_{-\infty}^{\infty} K(n(x-y)) f(y)\, dy.$$

   Prove that $\{f_n\}$ converges uniformly to f on $\mathbb{R}$.

2. Rudin, Chapter 7, #18.

Part II:

1. Rudin, Chapter 7, # 7.
2. Rudin, Chapter 7, # 9.
3. Rudin, Chapter 7, # 12.

On your own: Rudin, Chapter 7, #1-6 and the following problem:

Let $\{\phi_n\}$ be a sequence of nonnegative Riemann integrable functions on $[-1, 1]$ which satisfy
(i) $\int_{-1}^{1} \phi_n(t)\, dt = 1$ for each $n = 1, 2, 3, \ldots$
(ii) For every $\delta > 0$, $\phi_n \to 0$ uniformly on $[-1, -\delta] \cup [\delta, 1]$

1. Show that if $f : [-1, 1] \to \mathbb{R}$ is Riemann integrable and continuous at $x = 0$, then

$$\lim_{n\to\infty} \int_{-1}^{1} f(t)\phi_n(t)\, dt = f(0)$$

2. Show that

$$\lim_{n\to\infty} n \int_{-1/n}^{1/n} e^{-x^2}(1 - n^2x^2)\, dx = \frac{4}{3}$$

Reading: Rudin, Chapter 7.