STAT453/553, HW2, due Tuesday, Feb 21

1. [4 points] Use a computer to simulate samples of size $n = 3, 10, 50, 100$ from an exponential distribution with expected value 1. How does the correlation between the sample mean $\overline{X}$ and sample standard deviation S^2 depend on n? You can modify the code at

`https://math.unm.edu/~james/2023-STAT553/Independence.pdf`

to help answer the question. Give your answer without showing any code. You can explain what the correlations are without showing your code.

2. [4 points] Let X and Y be independent exponential distributions with $E[X] = \lambda$ and $E[Y] = \mu$, where $\lambda > \mu$. Find the density of $U = X + Y$. Note that for independent exponential random variables with the same mean, the sum has a gamma distribution. However, if they have different means, then the sum does not have a gamma distribution. (Refer to bivariate transformations in chapter 4 for how to do this problem).

3. Let $X_1, \ldots, X_n$ be an iid sample from a shifted exponential density:

$$f(x|\theta) = e^{-(x-\theta)} I(x > \theta)$$

(a) [4 points] Find the density of $X_{(1)}$ (the minimum observation)

(b) [4 points] Find the density of $X_{(n)}$ (the maximum observation)

(c) [4 points] Find the density of $X_{(2)}$ assuming that $n > 2$.

4. [4 points] Section 5.5 of the book gives a proof that for an iid sample of Uniform (0,1) random variables, the maximum $X_{(n)}$ converges in probability to 1 as $n \to \infty$. Show that $X_{(1)}$ converges in probability to 0.