

HW4, for MATH441, STAT461, STAT561, due October 23th

1. Consider rolling a 6-sided die and a 4-sided die, where the two dice are independent. Let X be the value of the 6-sided die minus the value of the 4-sided die.

- (a) Determine the probability mass function for X .
- (b) Determine $E[X]$.
- (c) Determine $Var(X)$.

2. Let X be the value of a single fair die with n sides. For (a) and (b), assume the sides are labeled $1, 2, \dots, n$.

- (a) Suppose $n = 4$. Find $E[X]$ and $Var[X]$.
- (b) Recall the formulas $\sum_{i=1}^n i = \binom{n+1}{2}$ and $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$. Find $E[X]$ and $Var(X)$ for general n .
- (c) Suppose the sides of the die are labelled $m, m+1, m+2, \dots, m+n-1$. Find $E[X]$ and $Var(X)$. Hint: recall the properties of expectation and variance that $E[aX+b] = aE[X]+b$ and $Var(aX+b) = a^2Var(X)$.

3. Consider a game in which the probability of winning is 0.45 (for example, if you are playing at a casino, and the casino has probability 0.55 of winning). Suppose that to play the game you pay \$2. If you lose, then you lose the money. If you win, you get the money back and win an additional \$2. In other words, each time you play, you either lose \$2 or win \$2. Consider the following strategy for playing the game: You play the game once, and if you win, then you quit. If you lose, then you play two more times regardless of the outcome of the second game. Let X be your total winnings. Determine the probability mass function for X and $E[X]$.

4. Suppose the number of raisins in a cinnamon raisin bagel is Poisson with mean 6. Suppose you cut a bagel in half at a random location. (Assume that you don't cut through a raisin.)

- (a) What is the probability that your half-bagel has no raisins?
- (b) What is the probability that your half bagel has at least 3 raisins?

5. Let the cdf of a random variable X be given by

$$F(a) = \begin{cases} 0 & a < 0 \\ \frac{a}{3} & 0 \leq a < 1 \\ \frac{1}{3} + \frac{a-1}{4} & 1 \leq a < 2 \\ \frac{3}{4} & 2 \leq a < 3 \\ 1 & a \geq 3 \end{cases}$$

- (a) What is $P(X \leq 2)$?
- (b) Find $P(X = i)$, $i = 0, 1, 2, 3$
- (c) Find $P(\frac{1}{3} \leq X \leq \frac{3}{4})$. Note that generally for $a < b$, $P(a < X \leq b) = F(b) - F(a)$. Note that for a discrete random variable, $P(a \leq X \leq b)$ is usually not the same as $P(a < X \leq b)$. In other words, pay attention to strict versus not strict inequalities.
- (d) Plot $F(a)$ from $a = -1$ to $a = 4$.

6. I drive an unreliable car with electrical problems. The number of days, X , until the next breakdown for my car follows a geometric distribution with $p = 0.1$.

(a) Find the probability that my car doesn't break down over the next 10 days. I.e., find $P(X > 10)$.

(b) Given that my car doesn't break down in 10 days, find the probability that it doesn't break down the following day. I.e., find $P(X > 11|X > 10)$.

7. An unreliable car breaks down according to an exponential distribution with rate $\lambda = 1/1000$ (meaning an average of one breakdown per 1000 miles). Find

(a) The probability that it goes 2000 miles without breaking down, i.e., $P(X > 2000)$.

(b) The probability that it goes another 2000 miles without breaking down given that it has already gone 2000 miles without breaking down, i.e., $P(X > 4000|X > 2000)$.

8. Let X be a random variable with density

$$f(x) = \begin{cases} \frac{k}{x^4} & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

(a) $P(X > 2)$

(b) $E(X)$

(c) $E(X^2)$

(d) $E(X^3)$

(e) $F(x)$.

(f) $P(X > 3|X > 2)$