HW 4. Due Nov 6th.

1. Let X have density

$$f(x) = \begin{cases} kx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find k.

(b) Find $E[X]$

(c) Find $Var(X)$

(d) Find $F(x)$ and define this for all x in $(-\infty, \infty)$

(e) Find the density of $Y = \log X$.

2. Let X be an exponential random variable with mean β. Find the density for $Y = \log X$. Be sure to specify the density for the entire real line.

3. Let the amount of rain on rainy days be normally distributed with a mean of 3 cm and standard deviation of 1cm.

(a) Find the probability that there is at least 2cm of rain on a randomly chosen rainy day. You can use a normal table to get the probabilities needed or use `pnorm()` in R. Do get a final answer that is numerical and not just a function of $\Phi()$, the standard normal cdf.

(b) Suppose on rainy days the rate of a car accidents at a given intersection is .2 per day, and on non-rainy days, the rate of car accidents at the same intersection is .1 per day. If the probability of a rainy day tomorrow is 0.3, what is the probability that there will be no car accidents tomorrow at the intersection.

(c) Using the same information as (b), suppose there is exactly one car accident at the intersection on a particular day, but you don't know whether it was rainy that day (maybe you are visiting Australia...). What is the probability that it was raining that day?

4. For this problem, you'll compare the hypergeometric and binomial distributions. Suppose there is a sock drawer with N socks, each placed loosely in the drawer (not rolled into pairs). The total number of black socks is m. You take out a random sample of $n < m$ socks. Assume all the socks are the same shape, size, etc. and that each sock is equally likely to be chosen.

(a) Suppose the sampling is done without replacement. Calculate the probability of getting at least 2 black socks (your goal in order to wear matching black socks that day...) under the following conditions:

(i) $N = 10$, $n = 4$, $m = 5$.

(ii) $N = 20$, $n = 4$, $m = 10$.

(iii) $N = 40$, $n = 4$, $m = 20$.

(b) Suppose the sampling is done with replacement (this doesn't make much sense if you are planning to wear the socks!). Calculate the probability of getting at least two black socks when you sample four socks and the proportion of black socks is 0.5. Compare your answer to those in (a).

Problem 5 is for graduate students.

5. (Based on a true story.) Consider the very interesting previous problem about socks. Suppose I have 10 socks, 5 of which are black, and 5 of which are brown. Suppose I pick two socks out at a time (sampling without replacement), and each one forms a pair. So I just form 5 pairs of socks at random without worrying about matching. What is the expected number of matches?