HW5, for MATH441, STAT461, STAT561, due December 6th

1. Let X and Y be independent exponential random variables with mean 1. For all parts below, be sure to specify $f_Z(z)$ over the entire real line.

(a) What is $P(X > 2Y - 1)$?

(b) Find the distribution of $Z = X - Y$. Hint: You can use the convolution approach with

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(y) f_X(z + y)\, dy$$

(c) Find the distribution of $Z = X/Y$. Hint: You can use the convolution approach with

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(y) f_X(zy)\, y\, dy$$

For graduate students

(d) Derive a convolution-like formula for $Z = XY$ and use it to find $f_Z(z)$.

2. Let X be a uniform(0,1) variable. Let $Y|X$ be an exponential variable mean $1/X$.

(a) Find $E[Y]$

(b) FInd $Var(Y)$

(c) Write down a formula for the joint density of X and Y, $f_{X,Y}(x, y)$.

For graduate students:

(d) Find the marginal distribution of Y by integrating the joint density. You might need integration by parts or you can use a table of integrals.

3. Let X and Y be independent and identically distributed where X has density $f_X(x) = 2x^{-3} I(x > 1)$. Let $U = X/Y$ and $V = X$.

(a) Find the joint density of U and V.

(b) Find the marginal density of U.

(c) Find the marginal density of V.

(d) Find the conditional density $f_{U|V}(u|v)$.