Practice Final

Formula Sheet

$$Var(X) = E(X^2) - (E(X))^2, Cov(X,Y) = E[XY] - E[X]E[Y]$$

$$Cov\left(\sum_i X_i, \sum_j Y_j\right) = \sum_i \sum_j Cov(X_i, Y_j)$$

$$E\{E[X|Y]\} = E[X]$$

$$Var(X) = E\{Var(X|Y)\} + Var(E[X|Y])$$

The formula sheet will also include pdfs and pmfs for densities, so these don't have to be memorized.

1. (a) Suppose I roll a 6-sided die. If I get a six, then I roll again and keep rolling until I get something other than a six. If get something other than a six, I stop. Let X be the result. Show that $P(X = i) = 1/5$ for $i \in \{1, 2, 3, 4, 5\}$ and $P(X = i) = 0$ otherwise.

(b) Suppose I roll a four sided die and a six sided die and that both dice are fair and independent. If the two rolls are identical, then I roll both dice again until I get distinct values. Let X be the value of the four sided die and Y the value of the six sided die. Find the marginal distributions for X and Y. Also find $P(X < Y)$.

(c) Under the conditions of (b), what is $P(X = 1|Y = 6)$?

(d) Are X and Y independent? Why or why not?

2. Let $A = \{1, 2, 3, \ldots, n\}$. How many subsets of A are there?

3. Suppose a class has 40 students, of whom 13 are undergraduate math majors, 12 are undergraduate statistics majors, and 15 of whom are graduate students. Suppose a subset of 5 students is chosen at random. What is the probability the subset has at least one of each type of student (undergrad math, undergrad stat, and graduate student)? You can give an expression in terms of binomial coefficients rather than a specific number at the end.

4. Suppose $Y|X \sim \text{Pois}(X)$ and, $X \sim \exp(\lambda)$ (i.e., X has rate λ). Find

(a) $E[Y]$

(b) $Var(Y)$

5. Suppose $X \sim \text{gamma}(\alpha_1, 1)$ and $Y \sim \text{gamma}(\alpha_2, 1)$ are independent gamma random variables. Find the density for $Z = \frac{X}{X+Y}$.

6. Suppose the time until my car breaks down is exponential with mean 10 days (i.e., $\lambda = 1/10$) and the time until my wife's car breaks down is exponential with mean 15 days. What is the probability that her car breaks down first?

7. (a) X be exponential with rate λ and let Y be uniform(0,1) with X and Y being independent. Let $U = X + Y$ and $V = X$. Find the joint distribution of U and V and the marginal distribution of U.

(b) Find $Cov(X, U)$

Solution. I think I meant $Cov(X, U)$ because usually I use $Z = X + Y$.

$$Cov(X, U) = Cov(X, X + Y) = Cov(X, X) + Cov(X, Y) = Var(X) + 0 = \frac{1}{\lambda^2}$$

8. If there is no snow, the rate of accidents at an intersection is Poisson with rate $\lambda = 0.1$ for an entire day. If there is snow, there rate is $\lambda = 0.2$. If the chance of snow tomorrow is 30%, what is the probability of at least one accident? For this intersection, if there is at least one accident on a given day, what is the probability that it was snowing that day?