HW2, due 3/22/19

1. Let $Y_1, \ldots, Y_n$ be an iid sample from a normal distribution with mean μ and variance μ. Find the maximum likelihood estimate for μ and the information in μ in the sample. Use this to give the asymptotic variance of $\widehat{\mu}$.

2. A certain bit of software is said to extend the duration of your laptop battery by a factor of α. (For example, if $\alpha = 2$, the software is supposed to double the duration of the battery). To test this, n computers (with new batteries) are tested without the software and m are tested with the software. To model this, let $Y_1, \ldots, Y_n$ be iid exponential random variables with mean β, and let $Y_{n+1}, \ldots, Y_{n+m}$ be iid exponential random variables with mean $\alpha \cdot \beta$. Assume all $n + m$ random variables are independent.

(a) Write the likelihood and log likelihood functions.

(b) Find maximum likelihood estimators for α and β.

(c) Find the Information matrix for α and β.

(d) Give the approximate asymptotic variance for the value of $\widehat{\alpha}$ obtained in part (b).

(e) Design a simulation with $n = 10$, $m = 15$ and $\beta = 3$, $\alpha = 2$. Run the simulation at least 1000 times. Give a plot of $\widehat{\alpha}$ against $\widehat{\beta}$ and report the correlation between these two estimators.

(f) Use the theoretical information matrix to get the asymptotic variance for $\widehat{\alpha}$ and report how often the confidence interval derived from the information matrix captured the true value of $\alpha = 2$ in your simulation. For a CI, you can use

$$\widehat{\alpha} \pm 1.96 SE(\widehat{\alpha})$$

where $SE(\widehat{\alpha})$ is the square root of the $(1, 1)$ entry of the inverse of the information matrix for the sample.