HW1, STAT453/553, due 1/27/17

1. Let X be a random variable with density

$$f(x|\theta) = \frac{1}{2\theta}e^{-|x|/\theta}, \quad -\infty < x < \infty$$

(a) Find $E(X)$ and $Var(X)$ (you might need integration by parts to do this).

(b) Show that $f(x|\theta)$ forms an exponential family by putting it in the form used by Casella and Berger, and explicitly give the functions $h(x)$, $c(\theta)$, $w_1(\theta)$ and $t_1(x)$.

(c) Use Theorem 3.4.2 to find $E[|X|]$, the expected value of the absolute value of X.

(d) Confirm your answer in (b) by calculating

$$\int_{-\infty}^{\infty} \frac{|x|}{2\theta}e^{-|x|/\theta}\, dx$$

To calculate this integral recall that

$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

which is helpful for evaluating the integral piece-wise.

(e) For graduate students, also Use Theorem 3.4.2 to find $Var(|X|)$.

2. (a) Let X have a beta(a, a) distribution (i.e., normally we work with beta(a, b) distributions, but in this case let $b = a$. Put the density into the exponential family form and identify the functions $h(x)$, $c(\theta)$, $w_1(\theta)$ and $t_1(x)$.

(b) Let X have a normal distribution with mean μ and variance $\alpha\mu$ with $\mu, \alpha > 0$. For this problem, the variance is proportional to the mean but not equal to it for $\alpha \neq 1$. Consider μ and α to be the parameters of the distribution. Write the density in exponential family form and identify the functions $h(x)$, $c(\theta)$, $w_i(\theta)$ and $t_i(x)$, $i = 1, 2$.

3. If the high temperature in Albuquerque in January normally distributed with a mean of 45 degrees Fahrenheit with a standard deviation of 10 degrees Farenheit, what is the distribution of the high temperature in January measured in Celsius?

4. Consider the pdf

$$f(x) = 280x^4(1-x)^3 \quad I(0 < x < 1)$$

(a) Plot the pdf (you might use a computer to calculate enough points to make a reasonable plot).

(b) Now make a graph of $(1/\sigma)f((x-\mu)/\sigma)$ for the following values:
(i) $\mu = 0, \sigma = 2$
(ii) $\mu = 1, \sigma = 1$
(iii) $\mu = -2, \sigma = 3$

You can either sketch the graphs by hand or use a computer.