HW2, for MATH441, STAT461, STAT561, due September 9th

1. You flip a coin until you get tails. Describe the sample space. How many points are in the sample space?

2. Two five-sided dice are rolled. The dice can be distinguished. Write down the sample space.

3. Two five sided dice are rolled. Let A be the event that the sum is an odd number. (a) List the outcomes in A. (b) Determine $P(A)$.

4. In an ordinary deck of 52 cards, there are four aces (ace of spades, ace of diamonds, ace of clubs, ace of hearts). Suppose you are given 2 cards at random (without replacement) from a shuffled deck, and after your two cards are given, your friend is given two cards at random from the remaining 50 cards. Let A be the event that you have two aces. Let B the event that your friend has two aces. Compute (a) $P(A)$, (b) $P(B)$, (c) $P(AB)$, (d) $P(A \cup B)$. You can assume that by symmetry, $P(B) = P(A)$.

5. Two six sided dice are rolled repeatedly until the sum is either 2 or 3. Find the probability that a 2 is rolled first. Hint: let E_i be the event that a 2 is rolled on the ith round and that neither a 2 nor a 3 has occurred on any previous round. Let E be the event of interest. Then argue that the event $E = \cup_{i=1}^{\infty} E_n$ and compute $P(E) = P\left(\cup_{i=1}^{\infty} E_n\right)$.

6. Assume that the probability that a randomly selected person has a birthday in a given month is 1/12. In other words, the probability that a birthday occurs in January is 1/12, the probability that it occurs in February is 1/12, etc. What is the probability that in a group of 5 people, each person has a birthday in a distinct month? Hint: This is a variation of the famous birthday problem which asks the probability that a group of n people have distinct birthdays considering 365 possible birthdays.

7. In a certain population, 4% of people are color blind. A subset is to be randomly chosen. How large must the subset be if the probability of its containing at least one color blind person is at least 95%? (Consider the population to be large enough that you can consider it to be infinite, so the selection can be considered with replacement. In other words, don't worry about whether a random sample might sample the same person twice.)

8. Consider flipping a coin twice where the only possibilities are heads and tails. Let A be the event that both tosses result in the same value (both heads or both tails). Let B be the event that the second toss is heads. Find $P(AB)$ and $P(A \cup B)$.

9. Suppose $P(A^c) = 1/5$ and $P(B) = 1/4$. Can A and B be mutually exclusive? Why or why not?

10. Suppose 5 indistinguishable balls are placed at random into 5 distinguishable bins. Find the probability that exactly one bin is empty. Hint: First determine that probability that the first bin is empty.