HW2 for MATH441, STAT461, STAT561, due September 8th

1. You flip a coin until you get tails. Describe the sample space. How many points are in the sample space?

2. (a) Two four-sided dice are rolled. The dice can be distinguished. Write down the sample space.

(b) Now suppose that two four-sided dice are rolled that cannot be distinguished. Write down the sample space.

3. Two fair three-sided dice are rolled. (How do you make a three-sided die? Take a six-sided die. Divide the number by 2 and round up if it is not an integer.) Let A be the event that the sum is an odd number. (a) List the outcomes in A (you can assume the dice can be distinguished). (b) Determine $P(A)$.

4. In an ordinary deck of 52 cards, there are four aces (ace of spades, ace of diamonds, ace of clubs, ace of hearts). Suppose you are given 2 cards at random (without replacement) from a shuffled deck, and after your two cards are given, your friend is given two cards at random from the remaining 50 cards. Let A be the event that you have two aces. Let B the event that your friend has two aces. Compute (a) $P(A)$, (b) $P(B)$, (c) $P(AB)$, (d) $P(A \cup B)$. You can assume that by symmetry, $P(B) = P(A)$.

5. Two six sided dice are rolled repeatedly until the sum is either 6 or 7. Find the probability that a 6 is rolled first. Hint: let E_i be the event that a 6 is rolled on the ith round and that neither a 6 nor a 7 has occurred on any previous round. Let E be the event of interest. Then argue that the event $E = \cup_{i=1}^{\infty} E_n$ and compute $P(E) = P\left(\cup_{i=1}^{\infty} E_n\right)$.

6. In a certain population, one out of 25 people are color blind, on average.

(a) If a random sample of 30 people is chosen, what is the probability that there is at least one color blind person? Hint: Find the probability that none of the 30 people is color blind.

(b) Use the inclusion-exclusion formula to find the probability that in a random sample of two people (i.e., the people are independently chosen), at least one of the two is color blind.

(c) Use the inclusion-exclusion formula to find the probability that in a random sample of three people, at least one of the three is color blind.

7. Suppose $P(A^c) = 1/5$ and $P(B) = 1/4$. Can A and B be mutually exclusive? Why or why not?

8. (For graduate students).

(a) Consider rolling a fair six-sided die until a 6 appears. How many times do you have to roll the die so that the probability of at least one 6 is at least 90%?

(b) Determine a more general formula for an m-sided fair die with sides $1, 2, \ldots, m$ where you determine the number of rolls needed so that the probability of at least one m is at least p.

9. (For graduate students). Suppose 2 indistinguishable marbles are placed at random into 2 distinguishable bins.

(a) List all the possible arrangements of the marbles in the bins. (You can draw pictures if it helps.)

(b) Are all arrangements equally likely? Why or why not?