

HW3, for MATH441, STAT461, STAT561, due September 19th

1. Suppose $P(A) = 0.9$ and $P(B) = 0.8$. Is it possible that $P(AB) = 0.6$? Hint: Use Bonferroni's inequality.

2. Suppose A and B are independent with $0 < P(A), P(B) < 1$. Show that A and B^c are also independent.

3. Lead and bacteria are two common sources of contamination in a water distribution system. Suppose 4% of the water distribution systems are contaminated by lead, and only 2% of the water distribution systems are contaminated by bacteria. Assume that the events of lead and bacterial contamination are statistically independent.

(a) Determine the probability that a water distribution system selected at random for inspection is contaminated.

(b) If a system is indeed contaminated, what is the probability that it is caused by lead only?

4. According to past records, a batch of mixed concrete supplied by a certain supplier can be of high quality (H), medium quality (M), or low quality (L), with respective probabilities of 0.2, 0.7, and 0.1, respectively. Suppose that the probability of failure of a reinforced concrete component would be 0.001, 0.01, or 0.1 depending whether the quality of the concrete is high (H), medium (M), or low (L), respectively.

(a) What would be the probability of failure of a reinforced concrete structural component cast with a batch of concrete supplied by the manufacturer?

(b) A test may be performed to give more information on the quality of concrete supplied by the manufacturer. The probabilities of passing the test for high, medium, and low quality concrete are 0.90, 0.70, and 0.20, respectively. If a batch of concrete passed the test

(i) What is the probability that it will be of high quality?

(ii) In this case, i.e., concrete passed the test, what would be the probability of failure of a structural component cast from this batch of concrete?

5. Two players take turns throwing a basketball until someone makes a basket, who is called the winner. The probability that the first player makes a basket is 0.4. The probability that the second player makes a basket is 0.5. Assuming that the players take turns until someone makes a basket and that each throw is independent with probabilities not changing, what is the probability that the first player wins?

6. Suppose $P(A) = 0.6$ and $P(B) = 0.7$. Can A and B be mutually exclusive? Why or why not? Can they be independent?