HW3, for MATH441, STAT461, STAT561, due September 20th

1. Suppose A and B are independent with $0 < P(A), P(B) < 1$. Show that A^c and B^c are also independent.

2. Lead and bacteria are two commmon sources of contamination in a water distribution system. Suppose 4% of the water distribution systems are contaminated by lead, and only 2% of the water distribution systems are contaminated by bacteria. Assume that the events of lead and bacterial contamination are statistically independent.

(a) Determine the probability that a water distribution system selected at random for inspection is contaminated.

(b) If a system is indeed contaminated, what is the probability that it is caused by lead only?

3. According to past records, a batch of mixed concrete supplied by a certain supplier can be of high quality (H), medium quality (M), or low quality. (L), with respective probabilities of 0.2, 0.7, and 0.1, respectively Suppose that the probability of failure of a reinforced concrete component would be 0.001, 0.01, or 0.1 depending whether the quality of the concrete is high (H), medium (M), or low (L), respectively.

(a) What would be the probability of failure of a reinforced concrete structural component cast with a batch of concrete supplied by the manufacturer?

(b) A test may be performed to give more information on the quality of concrete supplied by the manufacturer. The probabilities of passing the test for high, medium, and low quality concrete are 0.90, 0.70, and 0.20, respectively. If a batch of concrete passed the test
(i) What is the probability that it will be of high quality?
(ii) In this case, i.e., concrete passed the test, what would be the probability of failure of a structural component cast from this batch of concrete?

4. Two players take turns throwing a basketball until someone makes a basket, who is called the winner. The probability that the first player makes a basket is 0.4. The probability that the second player makes a basket is 0.5. Assuming that the players take turns until someone makes a basket and that each throw is independent with probabilities not changing, what is the probability that the first player wins?

5. A bag contains 9 marbles, of which 3 are red and 6 are blue. Players 1 and 2 take turns drawing marbles from the bag without replacement until a red marble is drawn. (Assume all marbles in the bag are equally likely to be drawn each round.) For example, Player 1 draws a blue from the bag with 9 marbles, then Player 2 draws a blue from the bag with 8 marbles remaining, then Player 1 draws from the bag with 7 marbles remaining, etc. What is the probability that Player 1 is the first to draw the red marble?

6. (Graduate students). Suppose six fair, independent 6-sided are tossed. What is the probability of getting three pairs? Examples of tosses that result in three pairs include $(2, 2, 3, 3, 5, 5), (2, 6, 6, 4, 4, 2)$, etc. Assume that for the three pairs, there are three distinct values. I.e., $(2, 2, 3, 3, 3, 3)$ does not count as three pairs.

7. (Graduate students for both parts). (a) On my shelf I randomly place 10 books in row, two of which are probability textbooks. What is the probability that the two probability textbooks happen to be next to each other?

(a) A pearl necklace is made from n pearls, two of which are defective. What is the probability that the two defective pearls are next to each other? Here, assume that the order of the pearls is random (all orders are equally likely) and that the necklace has no first or last pearl. The pearls are just arranged on a circle.